GENERATION OF REGIONAL BUSINESS CYCLES THROUGH INTERREGIONAL FEEDBACK MECHANISMS

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Regional economic development is subject to effects of interregional economic interactions. In particular, a region with a <u>closed</u> economic growth can be unconditionally void of any tendency to business cycle oscillations. Such inherently non-oscillatory regions can be arranged into systems of simple interactions causing regional business cycles through <u>interregional</u> feedback mechanisms. If one aggregates all the economic activity for the whole interregional system, then it is possible that the information about the individual regional business cycles can be totally lost. Therefore, a non-oscillatory macroeconomic interregional behavior for a system of regions does not necessarily imply such non-oscillatory behavior for each of the regions involved in the system. On the contrary, one can show that such regional business cycles can, indeed, exist under non-oscillatory macroeconomic conditions for the system of regions. One can present a sequence of simple interregional cybernetic configurations with increasing multilateral complexity, exhibiting, respectively, increasing tendency for regional business cycles.

A Definition of Gross Regional Product

The Gross Regional Product (GRP), $Y_i(t)$ for a region R_i is defined as follows:

(1) $$Y_i(t) = C_i(t) + I_i(t) + E_i(t) - M_i(t)$$

In this expression

$C_i(t)$ is the total private and public consumption, including the depreciation of productive means,

$I_i(t)$ is the total private and public rate of formation of <u>new</u> productive means,

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$E_i(t)$ is the total export from the region R_i to the rest of the world, and

$M_i(t)$ is the total import into the region R_i from the rest of the world.

If

$$E_i(t) - M_i(t) = 0$$

then the economic behavior of the region R_i is characterized totally by intraregional determinants.

<u>The Simple Case of a Non-Oscillatory Intraregional Economic Development</u>.

For a simple case of regional economic development one assumes a linear consumption function.

(2) $$C_i(t) = b_i Y_i(t)$$

b_i is the marginal propensity to consume for the region R_i. Consistent with the assumption of a linear production function, the investment or rate of formation of new productive means is assumed to be as follows:

(3) $$I_i(t) = (1/r_i)\,\frac{dY_i(t)}{dt}$$

r_i is the regional rate of return on productive means [1, 2]. In a more complete treatment one could introduce various investment, consumption, import, and export lags [2]. In this treatment such lags are not assumed in order to assure, for sure, a non-oscillatory process of intraregional economic growth. Let

$$E_i(t) - M_i(t) = 0$$

Then, noting equations 1, 2 and 3, one obtains the following differential equation for regional economic development:

(4) $$\begin{cases} \dfrac{dY_i(t)}{dt} - r_i(1-b_i)\,Y_i(t) = 0 \\ Y_i(0) = Y_{i0} \end{cases}$$

The solution to this equation represents a non-oscillatory exponential growth of regional economic activity:

$$Y_i(t) = Y_{i0}e^{r_i(1-b_i)t} \quad (5)$$

In the subsequent treatment, such a non-oscillatory intra-regional economic development is assumed for all regions R_i, $i = 1, 2, \ldots, n$. That is, in this formulation it is impossible to generate intraregional business cycles. If regions are allowed to have interregional interactions, it is still possible to generate regional business cycles. But such oscillations are then generated by interregional feedback mechanisms.

A Unidirectional Multilateral Interregional Development Model

Figure 1 illustrates the topology for a unidirectional multilateral interregional trade model for n regions R_i, $i = 1, 2, \ldots, n$. It is assumed for simplicity that

$$\begin{cases} b_i = b, \ b \geq 0 \\ r_i = r, \ r \geq 0 \\ \alpha_i = \alpha, \ \alpha \geq 0 \\ 0 \leq \alpha + b \leq 1 \end{cases}$$

for all $i = 1, 2, 3, \ldots, n$. That is, all the n regions are behaviorally similar. α is the coefficient of proportionality of exports from the region R_i to the region R_{i+1}. $\alpha + b$ is the fraction of the GRP of the region R_i allocated to regional consumption and exports. This fraction must be non-negative and at most unity for a meaningful interpretation of economic activity. Then

$$E_i(t) = \alpha Y_i(t) \quad (6)$$

and

$$M_i(t) = \alpha Y_{i-1}(t) \quad (7)$$

These forms of exports and imports represent a special case. However, they seem reasonable and simple for the purpose of this investigation. They merely claim that exports from the region R_i into the region R_{i+1} are proportional to GRP of the region R_i. One could, indeed, assume other forms for such import and export relationships. For example, E_i could be proportional to the rate of change of GRP of the region R_i, or it could be assumed to be proportional to GRP of the region R_{i+1}, etc. [2]. For the purposes of this investigation, equations 6 and 7 are sufficient for the demonstration of the generation of regional business cycles by interregional feedback mechanisms.

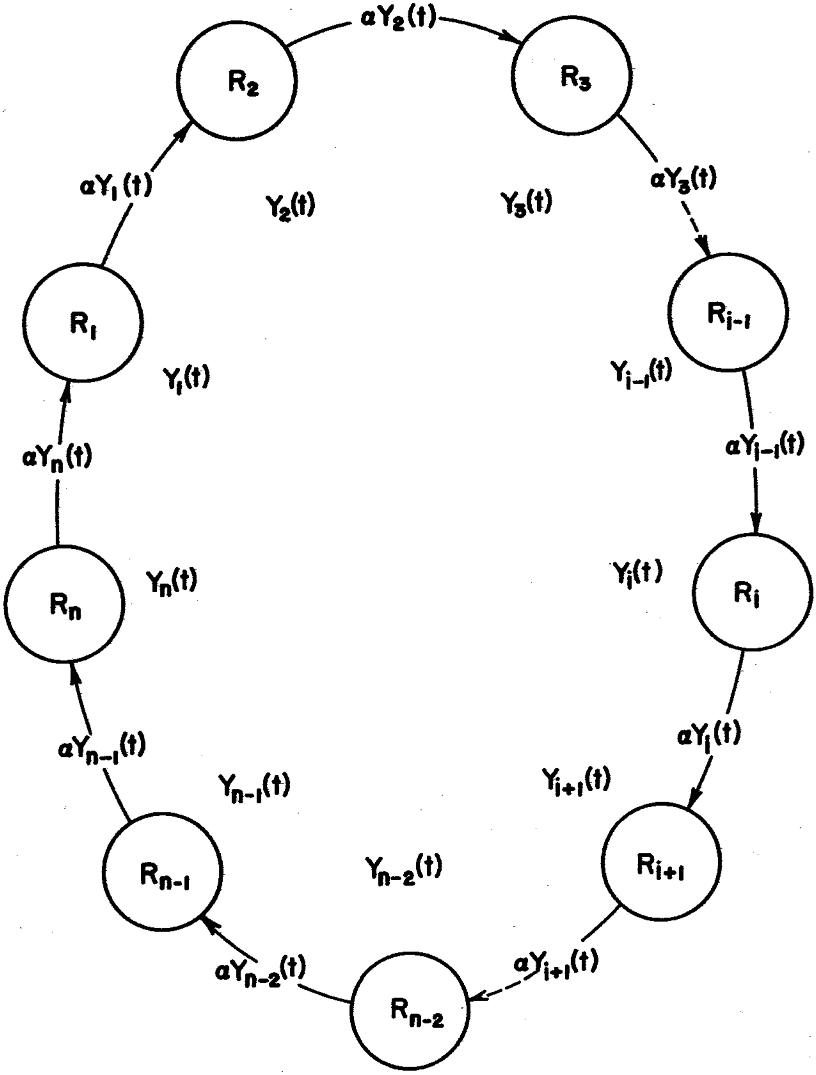
αY2(t)
R2
R3
αY1(t)
Y2(t)
Y3(t)
αY3(t)
R1
Ri-1
Y1(t)
Yi-1(t)
αYi-1(t)
αYn(t)
Rn
Yn(t)
Yi(t)
Ri
αYi(t)
αYn-1(t)
Yn-1(t)
Yi+1(t)
Rn-1
Yn-2(t)
Ri+1
αYn-2(t)
αYi+1(t)
Rn-2

Figure 1

The equation for the gross regional product, $Y_i(t)$ is then as follows:

$$(8)\quad \begin{cases} Y_i(t) = bY_i(t) + (1/r)\dfrac{dY_i(t)}{dt} + \alpha Y_i - \alpha Y_{i-1} \\ Y_i(0) = Y_{01} \\ \text{For } i = 1,\ i-1 = n;\ i = 1, 2, 3, \ldots, n. \end{cases}$$

This differential equation can be rearranged into the form:

$$(9)\quad \frac{dY_i(t)}{dt} - (1 - b - \alpha)\, rY_i(t) = r\alpha Y_{i-1}(t)$$

By definition,

$$(10)\quad A_0 = r(1-b) \text{ and}$$

$$A = r(1 - b - \alpha) = A_0 - r\alpha$$

A_0 is the intraregional rate of growth of economic activity. A is the regional rate of growth of economic activity when the particular multilateral interregional interactions described above are allowed.

Again, for simplicity, the following initial conditions are assumed:

$$Y_{10} > 0;\ Y_{02} = Y_{03} = \ldots = Y_{0n} = 0$$

The choice of these initial conditions makes region R_1 the source and initiator of the economic development for the whole interregional system. The economic activity of region R_1 migrates and diffuses in a manner of a neighbor-to-neighbor chain reaction to the remaining regions, and feeds eventually back into the source region R_1 itself.

By taking Laplace transform of equation 9, and after rearranging it, one obtains the following expression:[1]

$$(11)\quad \begin{cases} (s - A)\, \mathcal{Y}_i(s) - r\alpha \mathcal{Y}_{i-1}(s) = Y_{01} \\ Y_{01} > 0,\ Y_{02} = Y_{03} = \ldots = Y_{0n} = 0 \\ i = 1, 2, 3, \ldots, n \end{cases}$$

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[1] See Appendix.

In a matrix form, this system of equations appears as follows:

$$
(12) \qquad \begin{bmatrix}
(s-A) & 0 & 0 & \cdots & 0 & \cdots & -r\alpha \\
-r\alpha & (s-A) & 0 & \cdots & 0 & \cdots & 0 \\
0 & -r\alpha & (s-A) & \cdot & 0 & \cdots & 0 \\
0 & 0 & -r\alpha & \cdot & 0 & \cdots & 0 \\
 & & & & & & \cdot \\
\cdot & \cdot & \cdot & & & & \cdot \\
\cdot & \cdot & \cdot & & & & \cdot \\
\cdot & \cdot & \cdot & & & & \cdot \\
0 & 0 & 0 & \cdots & & -r\alpha & (s-A)
\end{bmatrix}
\begin{bmatrix}
y_1(s) \\ y_2(s) \\ y_3(s) \\ y_4(s) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_n(s)
\end{bmatrix}
=
\begin{bmatrix}
Y_{01} \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0
\end{bmatrix}
$$

The determinant of the matrix in this case can be shown to be as follows:

$$
(13) \qquad \Delta = (s-A)^n - (r\alpha)^n
$$

Since the only non-zero initial condition is Y_{01}, the appropriate cofactors needed are as follows:

$$
(14) \qquad \begin{cases} \Delta_{1i} = (-1)^{1+i} M_{1i} = (r\alpha)^{i-1}(s-A)^{n-i} \\ i = 1, 2, 3, \ldots, n \end{cases}
$$

The solutions for the Laplace Transforms of Gross Regional Products (GRP) are then

$$
(15) \qquad \begin{cases} y_i(s) = Y_{01} \dfrac{\Delta_{1i}}{\Delta} \\[2ex] \qquad\;\; = Y_{01} \dfrac{(\alpha r)^{i-1}(s-A)^{n-1}}{(s-A)^n - (\alpha r)^n} \\[2ex] i = 1, 2, 3, \ldots, n. \end{cases}
$$

Noting the shifting theorem of Laplace transforms, [3, 4] the inverse Laplace transform takes the form

$$(16) \qquad Y_i(t) = Y_{01} e^{At} \mathcal{L}^{-1} \left[\frac{(\alpha r)^{i-1} s^{n-i}}{s^n - (\alpha r)^n} \right]$$

For the characteristic equation

$$s^n - (\alpha r)^n = 0$$

there is, among other roots, the real root

$$s = \alpha r$$

Then one can show that

$$(17) \qquad s^n - (\alpha r)^n = (s - \alpha r) \sum_{i=1}^{n} (\alpha r)^{i-1} s^{n-i}$$

It can be now shown that if one aggregates the Gross Product over all the n regions R_i, $i = 1, 2, 3, \ldots, n$, then the details concerning business cycles of any one and all particular regions will be lost. Noting equation 17, equation 16 becomes as follows:

$$(18) \qquad Y_i(t) = Y_{01} e^{At} \mathcal{L}^{-1} \left[\frac{(\alpha r)^{i-1} s^{n-i}}{(s-\alpha r) \sum_{i=1}^{n} (\alpha r)^{i-1} s^{n-1}} \right]$$

The total gross product for the n regions is then

$$(19) \qquad Y(t) = \sum_{i=1}^{n} Y_i(t) = Y_{01} e^{At} \mathcal{L}^{-1} \left[\frac{\sum_{i=1}^{n} (\alpha r)^{i-1} s^{n-i}}{(s-\alpha r) \sum_{i=1}^{n} (\alpha r)^{i-1} s^{n-i}} \right]$$

$$= Y_{01} e^{At} \mathcal{L}^{-1} \left[\frac{1}{s-\alpha r} \right]$$

$$= Y_{01} e^{At} e^{\alpha r t} = Y_{01} e^{A_0 t}$$

where $A_0 = A + \alpha r$ by equation 10. Therefore, the whole system of regions will grow with the same characteristics as region R_1 would without any interregional interactions. To put this another way, in the unidirectional multilateral development system, the GRP of region R_1 is conserved but becomes "geographically" diffused and redistributed over all the n regions.

This process of diffusion and distribution includes also a feedback process from each region through the remaining regions back to itself. Even though the total aggregated Gross Product remains non-oscillatory, such interregional feedback effects can generate regional business cycles. The macroeconomics of the System of the n Regions will not suffice in the study of interregionally generated regional business cycles. One must preserve the structural aspects characterized by equation 16 for each particular region. That is, a regional resolution and disaggregation is necessary for the study of the local regional economic dynamics.

Regional Business Cycles versus Increasing Multilateral Complexity of Interregional Economic Interactions

The simplified unidirectional multilateral model introduced here can be used to demonstrate an increasing tendency to regional business cycles with increasing multilateral complexity of the interregional "ring" structure.

Example No. 1: The Unidirectional Bilateral Case. For a simple bilateral system the following relationship holds:

$$(20) \qquad \begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} s-A & -r\alpha \\ -r\alpha & s-A \end{bmatrix} \begin{bmatrix} Y_{01} \\ 0 \end{bmatrix}$$

The characteristic equation for this case is

$$\Delta = (s-A)^2 - (\alpha r)^2$$

The roots of Δ are

$$\begin{cases} s_1 = A + \alpha r \\ s_2 = A - \alpha r \end{cases}$$

Then

$$(21) \qquad \begin{cases} y_1(s) = Y_{01} \dfrac{s-A}{[s-(A+\alpha r)]\ [s-(A-\alpha r)]} \\[2ex] y_2(s) = Y_{01} \dfrac{\alpha r}{[s-(A+\alpha r)]\ [s-(A-\alpha r)]} \end{cases}$$

By taking the respective Laplace inverse transforms, one obtains the following solutions:

(22) $$\begin{cases} Y_1(t) = Y_{01}e^{At} \cosh (\alpha rt) \\ Y_2(t) = Y_{01}e^{At} \sinh (\alpha rt) \end{cases}$$

Further, one notes that

$$\begin{aligned} Y(t) &= Y_1(t) + Y_2(t) \\ &= Y_{01}e^{At} [\cosh (\alpha rt) + \sinh (\alpha rt)] \\ &= Y_{01}e^{At}e^{\alpha rt} = Y_{01}e^{A_0 t} \end{aligned}$$

Thus, the specific regional dynamics expressed by equations 22 are lost in the aggregation process. It is seen that the bilateral case does not generate regional business cycles. Each region has a non-oscillatory growth of economic activity.

Example No. 2: The Unidirectional Trilateral Case. Whereas the bilateral case was unconditionally non-oscillatory, the trilateral case will exhibit a definite tendency to regional oscillations of growing economic activity. The unidirectional trilateral system, where region R_1 is the initiating source of economic development, is characterized by the following relationship

(24) $$\begin{bmatrix} s-A & 0 & -r\alpha \\ -r\alpha & s-A & 0 \\ 0 & -r\alpha & s-A \end{bmatrix} \begin{bmatrix} y_1(s) \\ y_2(s) \\ y_3(s) \end{bmatrix} = \begin{bmatrix} Y_{01} \\ 0 \\ 0 \end{bmatrix}$$

The roots of the respective characteristic equation

$$\Delta = (s-A)^3 - (r\alpha)^3 = 0$$

are

$$\begin{cases} s_1 = A + r\alpha = A_0 \\ s_2 = [A - (1/2)r\alpha] + j \dfrac{\sqrt{3}}{2} r\alpha \\ s_3 = [A - (1/2)r\alpha] - j \dfrac{\sqrt{3}}{2} r\alpha \; ; \; j = \sqrt{-1} \end{cases}$$

The pair of complex conjugate roots δ_2 and δ_3 correspond to an oscillatory term. One can solve then, noting equations 13, 14, 15 and 16, for $Y_1(t)$, $Y_2(t)$ and $Y_3(t)$ corresponding, respectively, to the GRP of the regions R_1, R_2 and R_3:

$$(25)\quad \begin{cases} Y_1(t) = Y_{01}e^{A0t}\left[1/3 + (2/3)e^{-(3/2)r\alpha t}\sin(\sqrt{3/2}\ r\alpha t + \pi/2)\right] \\ Y_2(t) = Y_{01}e^{A0t}\left[1/3 + (2/3)e^{-(3/2)r\alpha t}\sin(\sqrt{3/2}\ r\alpha t - \pi/6)\right] \\ Y_3(t) = Y_{01}e^{A0t}\left[1/3 + (2/3)e^{-(3/2)r\alpha t}\sin(\sqrt{3/2}\ r\alpha t - 5\pi/6)\right] \end{cases}$$

It is seen that in this case the oscillatory terms are damped out rather fast, so that these oscillations do not have any strong tendencies to exhibit themselves in the regional behavior of economic development. Again, one can show that

$$Y(t) = Y_1(t) + Y_2(t) + Y_3(t) = Y_{01}e^{A0t}$$

whereby the aggregation destroys the detail of regional economic development and oscillations.

Example No. 3: The Unidirectional Quadrilateral Case. The quadrilateral case will show more tendency to pronounced oscillations of regional GRP than the trilateral case. One can apply again equations 12, 13, 14, 15 and 16. The roots of the characteristic equation

$$\Delta = (\delta - A)^4 - (r\alpha)^4 = 0$$

are

$$(26)\quad \begin{cases} \delta_1 = A + \alpha r \\ \delta_2 = A - \alpha r \end{cases}$$

$$\begin{cases} s_3 = A + j\alpha r \\ s_4 = A - j\alpha r \qquad , \; j = \sqrt{-1} \end{cases}$$

The roots s_3 and s_4 correspond to an oscillation, i.e., business cycle for each of the four regions in the system. The cofactors are

$$(27) \qquad \begin{aligned} \Delta_{11} &= (s-A)^3 & \Delta_{13} &= (\alpha r)^2 (s-A) \\ \Delta_{12} &= \alpha r (s-A)^2 & \Delta_{14} &= (\alpha r)^3 \end{aligned}$$

Using equations 12 through 16 and applying appropriate methods of inverse Laplace transforms, one obtains the following solutions for GRP's:

$$(28) \qquad \begin{aligned} Y_1(t) &= \tfrac{1}{2} Y_{01} e^{At} [\cosh(\alpha r t) + \cos(\alpha r t)] \\ Y_2(t) &= \tfrac{1}{2} Y_{01} e^{At} [\sinh(\alpha r t) - \sin(\alpha r t)] \\ Y_3(t) &= \tfrac{1}{2} Y_{01} e^{At} [\cosh(\alpha r t) - \cos(\alpha r t)] \\ Y_4(t) &= \tfrac{1}{2} Y_{01} e^{At} [\sinh(\alpha r t) + \sin(\alpha r t)] \end{aligned}$$

Again one can show that

$$Y(t) = Y_1(t) + Y_2(t) + Y_3(t) + Y_4(t) = Y_{01} e^{A_0 t}$$

where $A_0 = A + r\alpha$.

In this case the oscillations will show up much more than in the trilateral case. As the hyperbolic terms grow large in comparison to the bounded sinusoidial terms, the relative effect of oscillations or business cycles superimposed on the exponential growth of GRP will diminish as time goes on.

Example No. 4: The Unidirectional Octolateral Case. The roots of the characteristic equation of the octolateral case,

$$\Delta = (s-A)^8 - (r\alpha)^8 = 0$$

are as follows:

$$s_1 = A + r\alpha \qquad s_2 = A - r\alpha$$

$$s_3 = A + jr\alpha \qquad s_4 = A - jr\alpha$$

$$s_5 = (A + \frac{1}{\sqrt{2}} r\alpha) + j \frac{1}{\sqrt{2}} r\alpha \qquad s_6 = (A + \frac{1}{\sqrt{2}} r\alpha) - j \frac{1}{\sqrt{2}} r\alpha$$

$$s_7 = (A - \frac{1}{\sqrt{2}} r\alpha) + j \frac{1}{\sqrt{2}} r\alpha \qquad s_8 + (A - \frac{1}{\sqrt{2}} r\alpha) - j \frac{1}{\sqrt{2}} r\alpha$$

Therefore, the characteristic equation has the following specific form:

$$\Delta = (s-A)^8 - (r\alpha)^8$$

$$= \left[(s-A)^2 - (r\alpha)^2\right]\left[(s-A)^2 + (r\alpha)^2\right]\left[[(s-A) - \frac{\alpha r}{\sqrt{2}}]^2 + \frac{1}{2}(r\alpha)^2\right]$$

$$\left[[(s-A) + \frac{\alpha r}{\sqrt{2}}]^2 + \frac{1}{2}(r\alpha)^2\right]$$

Noting equations 15 and 16, the term exp At is extracted by Laplace Transform Shifting Theorem. The remaining denominator term in the Laplace transform to be inverted is as follows:

$$\left[s^2 - (r\alpha)^2\right]\left[s^2 + (r\alpha)^2\right]\left[(s - \frac{r\alpha}{\sqrt{2}})^2 + \frac{1}{2}(r\alpha)^2\right]\left[(s + \frac{r\alpha}{\sqrt{2}})^2 + \frac{1}{2}(r\alpha)^2\right]$$

Let K_0, K_1, K_2, K_3, K_4, K_5, K_6, and K_7 be appropriate constants. The term

$$[s^2 - (r\alpha)^2]$$

correspond to terms of the form

$$K_0 \cosh r\alpha t + K_1 \sinh r\alpha t.$$

The term

$$[s^2 + (r\alpha)^2]$$

corresponds to terms of the form

$$K_2 \cos r\alpha t + K_3 \sin r\alpha t.$$

The terms

$$\left[\left(s - \frac{r\alpha}{\sqrt{2}}\right)^2 + \frac{1}{2}(r\alpha)^2\right] \quad \text{and} \quad \left[\left(s + \frac{r\alpha}{\sqrt{2}}\right)^2 + \frac{1}{2}(r\alpha)^2\right]$$

correspond, respectively, to the time domain terms of the form

$$e^{\frac{r\alpha t}{\sqrt{2}}} \left[K_4 \cos \frac{r\alpha}{\sqrt{2}}t + K_5 \sin \frac{r\alpha}{\sqrt{2}}t\right]$$

and

$$e^{\frac{-r\alpha t}{\sqrt{2}}} \left[K_6 \cos \frac{r\alpha}{\sqrt{2}}t + K_7 \sin \frac{r\alpha}{\sqrt{2}}t\right]$$

It is noted that one of these oscillatory modes is an exponentially growing one. Thus, in this octolateral case one can generate exponentially growing oscillations or business cycles.

A Comparison of the Examples

One can now detect an important trend toward oscillatory behavior as one moves from the bilateral to trilateral, then to quadrilateral, and finally to octolateral case. As the degree of the multilaterality increases, the interregional feedbacks tend to generate increasingly oscillatory regional economic behavior. In fact, the octolateral case generated exponentially growing oscillatory modes. The quadrilateral case generated sustained oscillatory modes. The trilateral case generated exponentially decaying oscillatory modes, and the bilateral case was non-oscillatory.

Concluding Remarks

The simple example presented here suffices to establish the existence of the generation of regional business cycles by interregional feedback mechanisms. This simple example can be generalized several ways. For example, one might assume non-zero initial conditions for all the n regions in the unidirectional multilateral system of regions. It is also possible to introduce various investment and consumption lags which could make the intraregional economic behavior more susceptible to business cycles [2]. The unidirectional multilateral topology represents a particular form of a network of interactions. One could, indeed, introduce a great variety of other forms of interaction networks. Further, one could assume a great variety of different forms for import-export terms for the regions in the interregional economic system. All such generalizations and other ones could very well include the possibility of generating regional business cycles by interregional feedback mechanisms. The possibility of such generation of regional business cycles adds an important dimension to structural economic planning and policy processes.

APPENDIX

Consider a function of time $Y_i(t)$. Then the Laplace transform of $Y_i(t)$ is defined as follows:

$$\mathcal{Y}_i(s) = \mathcal{L}[Y_i(t)] = \int_0^\infty Y_i(t)e^{-st}dt$$

The inverse Laplace transform of $\mathcal{Y}_i(s)$ is as follows:

$$Y_i(t) = \mathcal{L}^{-1}[\mathcal{Y}_i(s)] = \frac{1}{2\pi j}\int_{a-j\infty}^{a+j\infty} \mathcal{Y}_i(s)e^{st}ds$$

a is chosen to the right of any singularity of $\mathcal{Y}_i(s)$; i = 1, 2, 3, ... , n. In particular, one notes the following properties of Laplace transforms:

1. $\mathcal{L}[kY_i(t)] = k\mathcal{Y}_i(s)$; k any constant

2. $\mathcal{L}[dY_i(t)/dt] = s\mathcal{Y}_i(s) - Y_{01}$

3. $\lim_{s\to\infty} s\mathcal{Y}_i(s) = Y_{01}$

4. $\mathcal{L}^{-1}[F(s-A)] = e^{At}\mathcal{L}^{-1}[F(s)]$

Comprehensive discussion on Laplace transform and inverse transform theories and properties are given in references [3] and [4].

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