ROLES OF FISCAL POLICY IN SPATIAL MACROECONOMIC DEVELOPMENT

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Traditionally, public policy is an important dimension of macroeconomic theory. In the so-called "dual" economies with a "private" and a "public" sector this policy is expressed through the latter sector, which is assumed to be powerful enough to influence effectively the former sector. For example, fiscal and monetary policies are designed to control the total economic system in some normatively desirable manner toward such goals as an adequate level of employment, a desirable rate of growth, an "appropriate" distribution of welfare, and the elimination of undesirable fluctuations in the economic activity. Most "open" economies are also concerned with the maintenance of appropriate export-import activities and the associated trade balances by some public policies. Directly or indirectly, through various production and consumption activities and the evolution of a human society, such dimensions as population dynamics, limitations of basic resources, generation of pollution and contaminants and their effect on human welfare, and the ultimately limited space for growth have been or need to be added to the domain of macroeconomic theory and its considerations of a public policy. A dual economy is a special case. For example, an economy could be totally public, and there can be several different types of dual economies. What is "good" or "bad" in this regard relates not only to some accepted social norms and traditions in a particular society, but to its ability to generate and sustain a life cycle of its own in a setting where it cannot, ultimately, be isolated from the "rest of the world". The purpose of this investigation is to explore some of these dimensions, especially through the means of fiscal policies, in a setting where space (e.g., land, distance, location) becomes an important explicit aspect of economic development in a "private-public" dual sector system. The background of this study is given in the references [1, 2, 3, 4] and the somewhat philosophical discussion provided in the reference [5].

Spatially Confined Growth: A Virtue or A Curse?

Saturation phenomena in economic development are getting new attention. Pollution, population explosion, limitations of natural resources, and, among other things, the spatial limits of a finite earth inspire ever new renovations

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of the older Malthusian pessimism about the ultimate fate of the mankind. In this regard, public policy cannot remain forever "neutral" or insensitive to the question of a desirable growth and life-cycle pattern of a society. "Locally," a spatially confined growth pattern is already a concern of urbanization processes. "Globally," the various public policies must be concerned with the ultimate finiteness of the "Mother Earth". Traditionally, growth has been synonymous with "progress". It has been used as a base for "moral justifications" for the ends in a Malthusian-Darwinistic framework of thinking. Spatial limitations represent an underlying aspect of any concentration phenomenon with some ultimate saturation limits. Therefore, space is and has been a scarce resource in such processes, for example, as the process of economic development.

There are several reasons why space is an important limiting resource in the process of an economic development, e.g., land, distance, location, transportation, spatial barriers, and upper limits of spatial concentration. Population density, pollution density, traffic density, and the density of a regional macroeconomic activity are spatial concentration phenomena with some ultimate upper limits. Spatial confinement is an important limiting factor for a process of growth. An unlimited growth concept is not a virtue, but rather a curse, since it is not really realizable in practice. As a process of a growth reaches a critical value, the result is often a serious and violent conflict requiring serious remedial aftermaths and recuperations. A public policy could take a preventive rather than a remedial attitude toward growth phenomena with critical saturation characteristics. The world need not be a Malthusian growth-death relaxation oscillator, although there are always those who like to see it that way. The role of public policy, and in particular, fiscal policy is not just that of maintaining an adequate employment and growth rate, redistribution of welfare, and the stabilization of the economic activity, but - in view of population trends - that of generating a desirable pattern of growth with appropriate limitations for a quality of life. This means, for example, a policy for population control, proper allocation of limited space to a limited concentration of economic activity, and conservation of limited natural resources such as plain fresh air.

A Process of Self-Destructive Growth

Consider a closed, spatially confined economic system. Let \( Y(t) \) be its gross regional product (GRP), which is equal to the population times the per capita gross regional product. Therefore, \( Y(t) \) measures indirectly also the population of this spatially confined society at a particular moment of time, \( t \). \( Y(t) \) is related to the pool of existing productive means, \( P(t) \), of the society (including labor, skills, health, natural resources, capital, etc.) by a linear production function [1, 2, 4]:

\[
(1) \quad Y(t) = r P(t); \quad I(t) = dP(t)/dt - (1/r) \left[ dY(t)/dt \right]
\]

\( I(t) \) represents the rate of formation of new productive means, or investment. The gross regional product allocated to the basic subsistence consumption, \( C(t) \) - which includes the depreciation of all productive means - is assumed
to be the fraction b of the total GRP:

\[ (2) \quad C(t) = b \cdot Y(t) \]

In the process of growth the society also loses or "consumes" its resources or productive means through depletion or consumption of factors necessary for sustained growth. Such depletion or consumption of "quality of life" will inhibit further growth. Examples of such inhibiting consumption of the "quality of life" are: growth of pollution and its growing impact on the health of people, depletion of natural resources and its inhibiting effect on production and growth of population, increasing social friction and losses of efforts due to increased density of economic activity (e.g., increasingly serious traffic jams, etc.), and the effects of excessively increasing population densities on public health due to increased concomitant incidents of psychological and social conflicts (crime, family disintegration, child abuse, detrimental use of drugs, etc.). Both, technological and social innovations can have effects on such inhibiting forces. Let \( X(Y(t)) \) represent this rate of loss of growth potential. For a process of self-destructive growth of a closed and spatially confined society it is further assumed that \( dX(Y)/dY \) is a monotonic increasing function of \( Y \).

The basic equation for the conservation of GRP is as follows:

\[ (3) \quad Y(t) = C(t) + I(t) + X(Y(t)) \]

By equation 2, the fraction \( b \) of \( Y(t) \) is allocated to the subsistence consumption while the remaining fraction \( 1-b \) is allocated to the investment in and the depletion of productive means. Noting equations 1, 2, and 3, one obtains the following differential equation:

\[ (4) \quad \frac{dY(t)}{dt} - r(1-b) \cdot Y(t) + rX(Y(t)) = 0 \quad Y(0) = Y_0 \]

The variables are separable, and, hence, one obtains the following integrated form:

\[ (5) \quad \int_{Y_0}^{Y(t)} \left[ Z - \left(1/(1-b)\right)X(Z) \right]^{-1} \, dZ = r(1-b)t \]

As a special illustration of a self-destructive growth process, assume that \( X(Y(t)) \) is of the form:

\[ (6) \quad X(Y) = \left[ \left(1-b\right)/Y_s \right] Y(t)^{n+1} \quad ; \quad n > 0. \]

\( Y_s \) is the doomsday saturation level of the society or a level at which the society is assumed to become unstable and self-destructive. An ecological
Example would be a yeast population in a finite wine keg. It grows and saturates, and dies as a result of either starvation (depletion of resources necessary for growth) or pollution (i.e., by the alcohol concentration produced by this population) or by a combination of both of these effects. Noting equations 5 and 6, \(Y(t)\) can be found to be:

\[
Y(t) = Y_0 \left[ 1 + \left( \frac{Y_0}{Y_S} \right)^n \left[ e^{nr} (1-b) t -1 \right] \right]^{-1/n} e^{r(1-b)t}
\]

Figure 1 illustrates the saturation characteristics of this expression for \(Y_0 = 1\) unit of GRP, \(r(1-b) = 4/100 = 4\) percent per annum growth rate, \(Y_S = 40\) units of GRP, and \(n = 1/2, 1\) and \(2\). When the nonlinearity of \(X(Y)\) is relatively mild, i.e., when \(n = 1/2\), the deviation from the unlimited exponential growth line occurs early, and the saturation is reached late in comparison to the case with \(n = 1\) or \(2\). For \(n = 2\), or more, the deviation from an unlimited growth pattern occurs late, and the saturation occurs early, i.e., symptoms occur late and the doomsday suddenly. From a public policy point of view, the case with \(n = 1/2\) allows considerably more planning time for preventive purposes than that with \(n = 2\). Therefore, the knowledge of the nature of \(X(Y)\) and its nonlinearity is very important for the prediction of the arrival at the doomsday. If the nature would replenish the depleted resources used up by the growth-saturation process leading to the self-destruction of the society, it is possible to imagine a repeated growth-saturation-death process so long enough stock is left over to initiate the subsequent cycle in the process. Such a sequence of "rises and falls of empires" is observed in several instances in nature, e.g., lemmings of the arctic regions. Such growth-death relaxation oscillations represent a type of "business cycle" phenomenon of an "animal kingdom". Figure 2 illustrates such a growth-death relaxation oscillation for equation 7 with \(n = 1/2\) and \(n = 2\). The death is assumed to be sudden when the saturation level is reached, where this level is at \(Y_S - \varepsilon\), \(\varepsilon\) being a very small fixed increment of \(Y_S\).

For Figure 2 the choice for \(Y_S - \varepsilon\) was about 33 units of GRP (see Figure 1) providing approximately 100 year cycle for \(n = 2\) and a 200 year cycle for \(n = 1/2\). It is assumed that \(Y_0\) is the same for all cycles, and is one unit of GRP. For the lack of a better name these kinds of growth-death relaxation oscillations may be called sopull (lemmus)* cycles after the observed behavior pattern of the arctic lemming societies, which commit a mass suicide when a critical population density is reached.

Models of Fiscal Policy for a Process of Spatially Confined Growth

It is possible to imagine several different kinds of dual economy systems and various roles of the public sector in such systems. The public sector is supposed to complement a private sector in such tasks as maintaining stability, growth, full employment of resources, distribution of welfare, and so on. The private sector is thought to be composed of "free" enterprises and "sovereign"

* sopull (Finnish), lemmus (Latin).
FIGURE 1: Patterns of Saturating and Potentially Self-destructive Growth for a Closed and Spatially Confined Society

Line of Unlimited Growth (exponential)

\[ Y_s = 40 \]

\[ n = 2 \]
\[ n = 1 \]
\[ n = 1/2 \]

GRP, \( Y(t) \)

time \( t \), years

40
FIGURE 2: Doomsday Relaxation Oscillations for $n = 1/2$ and $n = 2$ (illustration).

time in hundreds of years
consumers, maximizing on short run profits and satisfaction from economic goods, respectively. If the private sector is not willing or is not capable to do long run planning and allocation of resources for the survival of the society, then this role is assumed to be that of the public sector.

Especially in the spirit of the reference [5], the role of fiscal policy is that of the realm of "real economics" dealing with population, labor, capital and technology, education, land and space, and so on. Specifically, population control and its distribution over space and time is a matter which intersects the subject matter of fiscal policy. The role of population control is fundamental for the processes of spatially confined economic growth. For a finite region, when its gross regional product (GRP) increases, its gross regional product density (GRPD) measured as GRP/ area of the region, will eventually reach a critical level not desirable from the point of view of the survival of the society. In order to avoid such a doomsday saturation, the basic strategy will be a control of population to a level below the doomsday saturation. This is necessary but not sufficient. Among other things, also the per capita GRPs must be limited eventually to a level consistent with an 'acceptable' quality of life for all. As an example, a high level pollution associated with conspicuous consumption and the associated high level production activities can become so high that it overrides any possibility of an absorption through any feasible re-cycling.

Figure 3 illustrates the concept of a societal life-cycle planning for long-run survival. The idea is to limit the GRPD of a spatially confined society to an acceptable level y_p far enough below the doomsday level y_s so that the society has a chance to survive longer. This concept can be further illustrated in terms of a particular dual economy fiscal policy growth model.

Example No. 1

Consider a closed, spatially confined dual economy. Let \( Y \) be its GRP, \( C \) the private consumption, \( G \) the public consumption, \( I \) the private investment, and \( J \) the public investment. Then

\[
Y(t) = C(t) + G(t) + I(t) + J(t)
\]

Assuming a linear production function with a rate of return \( r \) on the sum of public and private productive means,

\[
I(t) + J(t) = (1/r) [dY(t)/dt]
\]

The private consumption is a fraction \( b \) of the disposable income \( Y(t) - T(t) \) where \( T(t) \) represents taxes by the public sector:

\[
C(t) = b[Y(t) - T(t)]
\]
FIGURE 3: An Illustration of the Planned Versus Unplanned Process of Growth Versus the Long-run Survival of a Spatially Confined Society

Doomsday Level of GRPD

Unplanned Path of Growth

Planned Level of Saturation

Planned Path of Growth

$y_0 = \text{initial GRPD}$

$y_p = \text{planned level of GRPD}$

$y_s = \text{doomsday saturation level of GRPD}$

time $t$ in appropriate units
Public consumption is assumed to be a fraction $g$ of $Y(t)$ plus a population depletion term which is assumed to be proportional to a power of $Y(t)$:

\[(11) \quad G(t) = g \cdot Y(t) + k \cdot Y(t)^{n+1}; \quad n > 0\]

Public investment is of the form:

\[(12) \quad J(t) = (h - g) \cdot Y(t)\]

Since the taxed real product $T(t)$ is completely allocated to the public consumption and investment,

\[(13) \quad T(t) = h \cdot Y(t) + k \cdot Y(t)^{n+1}\]

The total public and private consumption is then $C(t) + G(t) = [b \cdot (1-h) + g] \cdot Y(t)$, representing also the effects of redistribution of income from the private sector through the public sector by trading $b$ to $g$. Assume further that the society agrees to fix the per capita GRP to $p(t) = p_0$, so that

\[(14) \quad Y(t) = N(t) \cdot p(t) = N(t) \cdot p_0\]

where $N(t)$ is the population at the time $t$. The coefficient $k$ in the public consumption function can be defined as follows:

\[(15) \quad k = [(1 - b(1-h) - g) / [(1-b)N_p^n/p_0^n]]\]

$N_p$ represents the upper ceiling of population set by the public policy. Let the initial population be $N(0) = N_0$. Then, using the above equations, one can show that

\[(16) \quad N(t) = \left\{1 + (N_0/N_p)^n \cdot e^{r[1-b(1-h)-g]nt} \right\}^{-1/n} \cdot N_0 \cdot e^{r[1-b(1-h)-g]t}\]

Figure 4 would be an appropriate illustration of the effect of $n$ on the pattern of growth. The public policy parameters affecting the growth pattern are $h$, $g$, $n$ and $N_p$.

Example No. 2

Suppose the policy makers cannot come to an agreement on a finite $N_p$. In fact, assume $N_p$ goes to infinity. Then equation 16 reduces to the form

\[(17) \quad N(t) = N_0 \cdot e^{r[1-b(1-h)-g]t}\]

This growth pattern would reach a value $N_p$ at the time

\[(18) \quad t_p = [1-b(1-h)-g]^{-1} \ln \left(\frac{N_p}{N_0}\right)\]
FIGURE 4: Cross Section of a Radially Symmetric Spatial Pattern for a GRPD Limited Process of Economic Development
At this point of time, if at all feasible, the public policy could call for a value of \( h' \) and \( g' \) such that \( 1 = b(1-h') + g' \), i.e., all product would be consumed none being saved for the formation of new productive capacity, including population. Such a sudden choking would correspond to a limiting case of equation 16 where \( n \to \infty \) for \( N_p = N'_p \).

Example No. 3

Consider a special case of an open spatially confined economy. It can limit its growth by exports policies in addition to the other possibilities discussed in the previous examples. If equation 8 is modified to the form

\[
Y(t) = C(t) + G(t) + I(t) + J(t) + E(t)
\]

and the export process would belong to the public domain, being equal to the fraction \( \alpha \) of \( Y(t) \),

\[
E(t) = \alpha Y(t)
\]

then the public investment would be of the form

\[
J(t) = (h-g-\alpha) Y(t)
\]

If all the other assumptions are kept the same as in Example No. 1, then the equation 16 would still hold with the modification that the rate of growth be replaced by the expression \( r[1-b(1-h)-g-\alpha] \). The fiscal parameters would be now \( h, g, \alpha, n \) and \( N_p \). The growth of the system could be controlled, for example, by exporting all excess GRP or population into other regions.

Since \( Y(t) = N(t) p(t) \) where \( N(t) \) is the population at the time \( t \) and \( p(t) \) is the respective per capita GRP, the Malthusian argument could be generalized as follows: as \( N(t) \) grows without an adequate ceiling, then, eventually, \( p(t) \) will drop below a subsistence level. The quality of life can be increased to a point by increasing \( p(t) \). This can be done by holding population growth down in favor of increased productivity growth. But as per capita consumption and production increase, such effects as pollution can become growth limiting and self-destructive forces for the society.

GRPD Limited Patterns of Space-Time Growth

An important aspect of public policy is the spatial pattern of gross regional product density. In fact, one could specify a particular GRPD limited pattern of space-time growth within which a process of economic development must take place. The assignment of such GRPD limited patterns can have complications of economic mobility and transportation characteristics, nature and type of land usage and geographic distribution of natural resources, social and cultural characteristics of a particular society, and so on. Keeping in mind such possible important
factors of consideration, the purpose of this presentation is to keep the discussion in a framework of a spatial macroeconomic development. For the purpose of simplicity, only exponentially growing cases are considered; and a radially symmetric "center place" in an infinite isotropic and homogeneous plane is assumed. The radius of the center place is \( R_0 \), and the policy limited GRPD as a function of the radius \( R \) is some prescribed function of \( R \), \( y_p(R) \), and is assumed to be bounded and integrable over \( 0 \leq R \leq \infty \). Figure 4 illustrates a cross sectional aspect of this setting. The model presented here is assumed to behave as follows: first the center place area will grow to its policy limited level of GRPD. Then it spills over to its surrounding areas filling them up to the policy allowed limit \( y_p(R) \) as it moves radially outward. Let \( A(t) \) be a policy prescribed rate of growth of the economy as a function of time. Let \( Y(0) = Y_0 \) representing the "initial settlement". Then

\[
Y(t) = y_0 \exp \left[ \int_{0}^{t} A(\theta) \, d\theta \right]
\]

(22)

If the center place fills up to the maximum allowable level of GRPD in time \( t_f \) so that \( y_p = Y(t_f) / \pi R_0^2 = y_p / \pi R_0^2 \) represents a fixed ceiling over the area of the center place, then the surrounding areas will fill in as described by the following equation:

\[
Y_p + \int_{R_0}^{R(t)} y_p(R) 2\pi R \, dR = Y_0 \exp\left[ \int_{0}^{t} A(\theta) d\theta \right]; \quad t \geq t_f
\]

(23)

It is clear that this process involves exports of excess development over space in a radially outward direction. In this case \( A(t) \), \( y_p(R) \) and \( Y_p \) are public policy determined constraints. One can then determine the position \( R(t) \) of the outward moving edge or front of the economic development and its velocity \( dR(t)/dt \). The area of the already developed space is \( \pi R(t)^2 \) at the time \( t \).

In order to illustrate how equation 23 works in a special case, assume \( A(t) = A \), a constant rate of growth, \( y_p \) is constant over \( 0 \leq R \leq R_0 \), \( y_p(R_0) = Y_p \), and

\[
y_p(R) = \frac{K_n}{R^n}; \quad R \geq R_0
\]

\( n \) must be greater than or equal to zero for a bounded \( y_p(R) \). The center place fills up at the time

\[
t_f = \left(\frac{1}{A}\right) \ln \left[\frac{Y_p}{Y_0}\right] = \left(\frac{1}{A}\right) \ln \left[\frac{y_p}{y_0}\right]
\]

Then

\[
Y_p = Y_0 \exp A t_f
\]

\[
K_n = R_o^{n-2} \left(\frac{Y_p}{\pi}\right)
\]
Using equation 23, one can perform the appropriate integration and solve for \( R(t) \). In the range \( 0 < n < 2 \) the economic development, i.e., GRP, can be unbounded in an infinite region even though the GRPD remains bounded. For \( n > 2 \) GRP becomes bounded even in the infinite region. For \( t \geq t_f \) the cases are:

a. Unbounded GRP, \( 0 < n < 2 \):
\[
R(t) = R_o \left[(1-(n/2)) \left[e^{A(t-t_f)} -1\right] + 1\right]^{1/(2-n)}
\]

b. Unbounded GRP, \( n=2 \):
\[
R(t) = R_o e^{(1/2) \left[\exp[A(t-t_f)] - 1\right]}
\]

c. Bounded GRP, \( n > 2 \):

The process of growth terminates at the time
\[
t_{max} = (1/A) \ln \left[n/(n-2)\right] + t_f \quad \text{with}
\]
\[
Y_{max} = Y_p + \left[n/(n-2)\right] Y_o
\]

In order to get some numerical feeling of the behavior of this simple model, consider a center place of a 10 mile radius. Let the growth rate be 4 percent per annum and the initial value \( Y_o = $1000000/yr \). If the center place is allowed to expand by a factor of \( \exp(8) = 2981 \), then it would expand to about three billion economy in 200 years. Thereafter it would spill over to its surrounding areas. After another hundred years it would grow into a 163 billion economy.

If \( n = 0 \), the radius of the developed area would be 74 miles and the respective area would be about 17,000 square miles. If \( n = 1/2 \), the radius would increase to 118 miles with a respective area of some 44,000 square miles. If \( n = 1 \), then the radius would be 278 miles with an approximate area of some 243,000 square miles. The higher \( n \) becomes, the more "rural" would be the surrounding area of the center place, and the larger would be the area over which the "suburbia" would spread.

In equation 23 \( Y_p, R_o, R(t), y_p(R), \) and \( A(t) \) represent major "policy" characteristics, given an initial condition \( Y_o \). If any four of these characteristics are given one can solve for the fifth one. Therefore, it is possible to try out a variety of space-time designs of a GRPD limited economic development. In particular, one could specify \( Y_p, R_o, R(t) \) and \( y_p(R) \) and then solve for the growth rate \( A(t) \) required to realize these requirements.

**Strategies of Growth for Spatial Macroeconomic Development**

Equation 23 and the subsequent illustration of its use can be developed
further to illustrate various strategies of growth in a setting of spatial macroeconomic development: given \( \gamma_p, \gamma_o, R_o, R(t) \) and \( \gamma_p(R) \), what should \( A(t) \) be? The center place has a radius \( R_o \). For the radially symmetric surroundings the GRPD constraint is assumed to be of the bounded form:

\[
\gamma_p(R) = R_o^{n-2} \left( \gamma_p/\pi \right)/R^n ; R \geq R_o
\]

where

\[
\gamma_p = \gamma_o \exp \left[ \int_0^{t_f} A(\theta) \, d\theta \right]
\]

These expressions can be substituted in equation 23 which, then, can be solved for \( A(t) \):

\[
A(t) = \left[ (2-n) R(t)^{n-1} (dR(t)/dt) /[R(t)^{2-n} - (n/2)R_o^{2-n}] \right] ;
\]

\[0 < n < 2 ; R \geq R_o ; t \geq t_f\]

One should note that this growth rate \( A(t) \) is directly proportional to the radial velocity \( dR(t)/dt \) of the movement of the development front. Two examples are now used to illustrate the use of this growth rate equation.

Example No. 1

If \( R(t) = R_o(t/t_f)^k \) and so \( dR(t)/dt = (kR_o/t_f)(t/t_f)^k \) then

\[
A(t) = \left[ (2-n)k t^{k(2-n)-1} /[t^{k(2-n)} - (n/2)t_f^{2-n}] \right]
\]

for \( k = 0 \) the growth rate must be zero. Further,

if \( n = 0 \) and \( k = 1 \) then \( A(t) = 2/t \);

if \( n = 1 \) and \( k = 1 \) then \( A(t) = [t - (1/2) t_f]^{-1} \);

if \( n = 1 \) and \( k = 2 \) then \( A(t) = 2t/[t^2 - (1/2)t_f^2] \).

For \( n = 0 \) and \( k = 1 \), if \( t_f = 200 \) years, then \( A(t_f) = 0.01 \) or 1 percent per annum. At \( t = 300 \) years \( A(t) \) drops to 0.0033, and so on.

Example No. 2

If \( R(t) = R_o \exp[B(t/t_f)] \) and \( dR(t)/dt = B R_o \exp[B(t-t_f)] \) then

\[
A(t) = \left[ (2-n)B/[1-(n/2) \exp[B(2-n)(t-t_f)] \right]
\]
If the growth rate \( B \) for \( R(t) \) is zero, so is also \( A(t) \) for all \( t \geq t_f \). Further, 

if \( n = 0 \), then \( A(t) = 2B \); 

if \( n = 1 \), then \( A(t) = B/[1-(1/2) \exp(-B(t-t_f))] \).

In the latter case, as \( t \to \infty \) then \( A(t) \to B \).

In both of the above examples the usage of space is obviously equal to \( \pi R(t)^2 \). If one is going to specify how space is used up with some prescribed pattern for the ceiling of GRPD in a process of growth, then it is necessary to control the growth rate of GRP by some means such as fiscal policy. The practical implications of such a policy is a very long run planning process. Such a planning process may not be amenable to a private sector motivated by short run opportunities and exploitation with immediate returns. Eventually it needs to be done by a strong enough "global" public sector.

Further Comments on GRPD Limited SMED

The particular illustrations of GRPD limited patterns of spatial macroeconomic development (SMED) discussed above are simplifications in several ways. Among other things they do not assume diffusion and transportation delays and losses ("frictions") that could slow down a process of SMED considerably. These latter effects are discussed in references [1, 2, 4, 7], and the way they can be treated computationally for discrete models of SMED is discussed in the reference [6]. In connection with continuous diffusion models [1] it was clearly demonstrated that the dynamic characteristics of a source of economic development has a pronounced effect on the space-time growth of economic development. This suggests immediately the role of a fiscal policy in providing appropriate dynamic characteristics and spatial locations for such driving sources. This is particularly important in relation to various criteria of a GRPD limited growth. Figure 5 illustrates the difference between two dynamic sources at the location \( x = 0 \) in inducing a process of SMED with two different spatial patterns of GRPD [1]. Relatively speaking, one of these sources produces a much more uniform distribution of GRPD than the other one. The possibilities for various fiscal policies are further increased if one considers hierarchical structures such as those discussed in the references [4, 7]. For example, a set of dynamically specified sources can be generated by taxation whereafter the sources can be exported or geographically redistributed into appropriate locations for new centers of spatial economic development. This would be a form of space-time redistribution of welfare. In this situation the public sector assumes the responsibility for the "relocation of industry".

Remarks on Countercyclical Fiscal Policy in SMED Processes

The countercyclical fiscal policy in a dynamic economic system can conceivably have several different modes affecting production, investment, consumption and export-import activities and their lags or leads. It could act through feedback or feedforward mechanisms, or it could act as a dynamically
FIGURE 5: Two Dynamic Sources Generating Equal Total Gross Regional Product but With Different Spatial Distribution of Gross Regional Product Density

SOURCE LOCATED AT X=0
Duration of Process is the same for Source A and Source B
specified source stimulating an economy in a compensatory manner. In all these cases the objective of such a fiscal policy would be the elimination of business cycles or oscillations of space-time economic activity (i.e., of GRPD and GRP). One could present innumerable examples of such countercyclical fiscal policies in a setting of SMED. For the purposes of this presentation only one particular illustration will be presented. This example relates directly to a previous investigation on space-time business cycles generated by interregional feedback mechanisms [3]. Such phenomena can be important in the cases of very rapid multiregional economic development processes. If such processes are initiated by a source of economic activity, then such a source can be dynamically so designed that it will cancel out undesirable business fluctuations for each and all regions involved in the economic development. The appropriate dynamic characteristics of the driving source can, indeed, be dictated by an effective fiscal policy with an appropriate "systems approach" to the overall problem of stabilization and growth.

The reference [3] is devoted to the discussion on how a unidirectional, multilateral system of n regions, forming a single loop feedback configuration, generates space-time business cycles. In a particular illustration presented in this study, all the n regions have identical coefficients of production functions, marginal propensities to consume, and the fraction of GRP exported to the next region. For the presentation here, it will be assumed that all the initial conditions of GRP, \( Y_1(0) = Y_2(0) = \ldots = Y_i(0) = \ldots = Y_n(0) = 0 \) over all the n regions \( R_i \), \( i = 1, 2, 3, \ldots, n \). That is, one deals here with a completely virgin economic development problem. Region \( R_1 \) is primed by a source \( F(t) \) of economic activity, which is specified by a fiscal policy for economic development of the n regions. Region \( R_1 \) then primes through exports region \( R_2 \), which, in turn, primes region \( R_3 \), and so on, until through a completed chain reaction region \( R_n \) finally feeds back into region \( R_1 \). In the spirit of the reference [3], one can then readily show in the Laplace transformed complex domain that

\[
Y_i(\delta) = r \frac{F(\delta)}{[\alpha(r)]^{i-1} (\delta-A)^{n-i}} \left[ (\delta-A)^n - (ar)^n \right]
\]

\( A = r(1-b-\alpha) \)

\( r = \) rate of return on productive means

\( b = \) marginal propensity to consume

\( \alpha = \) fraction of GRP exported from region \( R_i \) to region \( R_{i+1} \)

\( i = 1, 2, 3, \ldots, n; \ n+1 = 1 \) (periodicity of n)

Assume further that the Laplace transform of \( F(t) \) is of the form

\[
F(\delta) = N(\delta-A) / D(\delta-A)
\]

where the denominator \( D(\delta-A) \) is a polynomial of \( \delta-A \) with a degree \( k+1 \), and the numerator \( N(\delta-A) \) is a polynomial of \( \delta-A \) with a degree \( k \), \( k = 0, 1, 2, \ldots, m \).
Another requirement for an "economic realizable" $F(t)$ is that this function be non-negative for all $t \geq 0$. This requirement introduces some restrictions to the choice of coefficients for the numerator and denominator polynomials of $F(\Delta)$. The choice of a strategy for a fiscal policy depends on the existence of the undesirable oscillatory modes appearing in the characteristic equation of the remaining Laplace transform expression after applying shifting theorem in the process of inversion:

\begin{equation}
Y_1(t) = \mathcal{L}^{-1} \left[ \frac{N(\Delta)/P(\Delta)}{(\Delta)^{i-1}} \frac{1}{\Delta^n - (ar)^n} \right]
\end{equation}

$D(\Delta)$ is so selected that it has no undesirable complex conjugate pairs of roots for the generation of undesirable oscillations of economic activity. $N(\Delta)$ is chosen so that it cancels out any pairs of undesirable complex conjugate roots of the characteristic equation

\begin{equation}
\Delta^n - (ar)^n = 0
\end{equation}

One can incorporate also additional criteria to the selection and synthesis of $F(t)$ so long it remains economic realizable.

The procedure of synthesizing an appropriate economic realizable $F(t)$ can be illustrated by considering a specific example. For a unidirectional octalateral case [3] one finds

\begin{align*}
\Delta^8 - (ar)^8 &= \\
&= 0
\end{align*}

The first task at hand is to investigate the seriousness of the three pairs of complex conjugate roots appearing in this expression. Starting from the left, the first bracket corresponds to a pair of real roots associated with a hyperbolic growth term. The second bracket corresponds to a sustained sinusoidal oscillation with a constant amplitude. The relative importance of this term diminishes as the growth term associated with the hyperbolic term swamps its effect out. The third bracket represents an exponentially damped sinusoidal oscillation. Since it will die out, it probably needs no special consideration. The fourth bracket seems to be the only potential trouble maker. It corresponds to an exponentially growing oscillatory term with an exponential growth rate $ar/\sqrt{2}$ and an oscillation with a period of repetition equal to

\begin{equation}
T = 2\pi/ (ar/\sqrt{2})
\end{equation}

The seriousness of the rate of growth and the period of oscillation can be investigated for various values of $r$ and $a$: 53
<table>
<thead>
<tr>
<th>$r$ years$^{-1}$</th>
<th>$\alpha$</th>
<th>$\alpha r/\sqrt{2}$ years$^{-1}$</th>
<th>$T$ years</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.00707</td>
<td>888.71</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.07071</td>
<td>88.87</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.17677</td>
<td>35.54</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>0.3535</td>
<td>17.77</td>
</tr>
</tbody>
</table>

It is seen that the oscillation term does become serious if one deals with large rates of returns on productive means and large fractions of GRP as exports from region to region. For relatively large $r$ and $\alpha$ the source $F(t)$ of the system of these eight regions must have a long-run countercyclical fiscal policy built into it in order to cancel out this growing oscillatory mode of space-time fluctuations in the loop of these regions. Among several possibilities of synthesising a particular $F(t)$ one could pick a simple one:

$$F(t) = F_0 e^{-\xi t} \left[ \frac{\lambda}{(\delta-(\alpha r/\sqrt{2}))^2+(1/2)(\alpha r)^2}/[(\delta+\alpha)(\delta-b)] \right]$$

The parameters $a$ and $b$ must be chosen so that $F(t)$ remains economic realizable. $F_0$ is an appropriate amplitude factor for the SMED of the eight regions. With this choice the numerator $N(\delta)$ of $F(\delta)$ cancels out the undesirable oscillatory terms of the characteristic equation.

The term that remains to be inverted can be expanded in partial fractions to the form

$$\frac{F_1}{\delta} + \frac{F_2}{(\delta+a)} + \frac{F_3}{(\delta-b)}$$

where

$$F_1 = -(\alpha r)^2/ab$$
$$F_2 = [a^2 + \sqrt{2} a \alpha r + (\alpha r)^2]/[a(a+b)]$$
$$F_3 = [b^2 - \sqrt{2} b \alpha r + (\alpha r)^2]/[b(a+b)]$$

For a positive $a$ and $b$, let $F_1 + F_3 = 0$. This would be the case if

$$b = (\alpha r) [\sqrt{2} + (\alpha r/a)]$$

Using this value of $b$ for $F_2$ one finds $F_2 = 1$. Therefore, with these values of partial fractions one finds, after taking the inverse Laplace transform of the partial fractions expansion above, that the required countercyclical source could be as follows:
\[ F(t) = F_0 e^{At} \left[ \frac{[(\alpha r/a)/(\sqrt{2} + (\alpha r/a))]}{[e^{(\sqrt{2} + (\alpha r/a))t} - 1] + e^{at}} \right] \]

While the parameters \( A, r, \) and \( \alpha \) are determined by the system of eight regions to which \( F(t) \) must be matched, the parameters \( a \) and \( F_0 \) represent additional degrees of freedom for the policy makers over and above the policy goal of eliminating the growing oscillatory mode from the system of SMED. Thus, even in this simple case, the policy makers could add additional policy goals to the fiscal policy.

**Concluding Remarks**

For a society, growth as such is not a realistic final goal. If a society could free itself from a habit of incremental decision making into a process of long run planning, it could select a long-run goal of a desirable life cycle with a pattern of controlled growth. This would require "global" rather than "local" attitudes in a space-time framework. In a system of dual economy the notion of long-run goals and planning is particularly interesting: it would be realistically possible only with a strong public sector with strong long-run commitments in a reasonably stable and mature political environment. Ultimately, a cooperative "world public sector" is needed with a strength to control, for example, highly manipulating mobile multinational corporations. It is quite clear that these multinational corporations are seeking and gaining power outside the control of "sovereign nations" and their internal public policies. Ultimately, only an effective "world public sector" can control excessive exploitation of basic limited resources needed for long-run survival of the mankind.

Traditionally, fiscal and monetary policies in a dual economy have been viewed as remedial complementary mechanisms for the correction of imperfections arising from the operations of the private sector. With these corrections, so long there has been some kind of an average growth of GNP and per capita income with a reasonable stability, employment and redistribution of welfare, the private sector is assumed to be doing its job in the "interest of public good". This view of a dual economy is one of an incremental process with an image of an ultimate objective: growth of GNP and per capita welfare. In the perspective of societal and cultural life cycles this is a short run point of view. Growth is not an end in itself; it is an aspect of a life cycle. Its purpose and desirability becomes defined only if a desirable life cycle has been specified. But the concept of a life cycle requires a motivation and an attitude for very long-run planning. This a private sector may not be willing nor capable of doing, and there is no "invisible hand" assuring that its performance is a guarantee of a splendid future for humanity.

A notable aspect of the fiscal policy in spatial macroeconomic development is its rather long planning horizon. It, among other objectives, is concerned with various growth strategies within finite spatial confines. Cooperation rather than a competitive conflict would be required for a properly balanced growth in a setting of a finite "lebensraum". Population control is one of the main mechanisms to achieve a balanced life cycle for mankind. In order to do this effectively, one must cut across a myriad of economic, cultural,
racial, and political problems. One of the objectives of this investigation has been to illustrate by simple examples the processes of a limited growth in a spatial setting. Planning horizons for such processes can run into several hundreds of years. Can we realistically expect that the mankind is mature and patient enough for such a space-time perspective? Another way out could be through a violent conflict where most of the mankind would be eliminated in favor of few. An important dimension of a life cycle oriented fiscal policy is not just its countercyclical and growth planning aspect, but its potential counter conflict effect.
REFERENCES


