ASSESSING STATE RESERVE REQUIREMENTS: A COST-BENEFIT APPROACH

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Introduction

States are typically influenced by exogenous economic fluctuations which are generally beyond the control of their fiscal policies. Therefore, if states are to meet the challenge of complying with balanced budget requirements during periods of economic fluctuation, it is vital that they insulate themselves from the contingent impacts of cash flow deficiencies resulting from such fluctuations. The recession from the fall of 1974 to the spring of 1975 stressed, on the one hand, the need to reevaluate financial reserve policies by accentuating the repercussions which could result when state governments fail to maintain adequate reserves. On the other hand, a number of states have a more than adequate financial reserve which represents, in part, an unnecessary tax burden on the residents of the state. Given such implications, the present paper addresses the question of annually estimating the adequate level of state reserves.

The question of adequacy of state reserves is considered through two factors in this paper. First, from a theoretical standpoint, a criterion is specified for measuring what reserves should be needed. Unfortunately, only limited study has been devoted to the area of state reserve holdings. However, works in the area of international reserve holdings by Heller [9], Hipple [12], Clark [2, 3], and others provide a useful framework for an inquiry of this kind. Heller and Hipple principally rely on a cost-benefit framework for assessing the optimum level of international reserves. In the present paper, we suggest a similar method for analyzing optimum state reserve holdings. The second part of the paper addresses the empirical questions involved in determining whether adequate reserves are being held by the various states. In considering these questions, a comparison of the estimated optimum level

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1In general, two approaches exist for determining the optimum level of international reserve holdings: the Heller-Hipple Cost-Benefit Approach and the Kelly-Clark Utility Maximization Method. The methodology of the latter group is one primarily of maximizing a quadratic (Kelly) or linear (Clark) utility function subject to a risk-yield opportunity constraint. The preference here, though, is to employ the former technique. See Kelly [15], Heller [9], Hipple [12], and Clark [2].
of reserves, $R^*$, and the actual level, $\bar{R}$, by an index, $R^*/\bar{R}$, is a useful way to test the adequacy of reserves. An example is shown to demonstrate how this index provides a useful test. Future research involving specific state data could utilize a similar methodology.

Optimum Level of State Reserves

State reserves, $\bar{R}$, can be defined as the cumulative residual of revenues over expenditures carried forward from year-to-year (i.e., from fiscal year $t-1$ to fiscal year $t$), plus excess revenues accumulated during the present fiscal year $t$. More specifically, reserves result from either a planned or unplanned surplus that is carried forward from a previous time period, or alternatively, by a surplus in the present period. Either source provides a working reserve.

The cumulative total of past and present period-generated reserve amounts should henceforth be thought of as the total reserve stock for a state. From reviewing state budget documents, it appears that states are generally reluctant to specify significant reserves for the projected budget period. Moreover, such stated reserves may not account explicitly for "carry forwards" accumulated during the last period, but not realized until after the revenue projection for time period $t$ was made. Nevertheless, this concept of reserves appears relevant for management purposes.

Given this definition, the first task is to delineate the optimum level of financial reserve a state should hold. To do this, three parameters need specification: the cost of holding state reserves, the benefits, and the probability that a reserve unit will actually be needed.

Each dollar utilized as a reserve entails a cost measured by the opportunity lost by taking such investment funds away from the public. Specifically, liquid reserves held by a state represent, in part, a capital resource of the state. Thus, the return lost by taking such investments away from the residents represents the marginal holding cost.

Meanwhile, a benefit can be realized in maintaining a certain reserve level. Such benefits can be estimated by assessing the adjustment cost avoided by holding reserves. Such cost avoidance includes the negative state income impact of fiscal policy changes such as tax increases or expenditure decreases required if reserves are unavailable to meet contingencies. Such costs may be avoided by having a stock of reserve assets available to meet temporary flow of funds problems or to offset revenue fluctuations. This conceptual framework provides the basis for defining reserve benefits.

Finally, the extent to which a state will actually need reserves is dependent upon: (1) the number of variances in revenue and expenditure patterns, and (2) the actual size of such fluctuations.

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2Both Heller and Hipple use an index to test the adequacy of international reserve. See Hipple [12, pp. 632-34], and Heller [9, pp. 309-11].
Given these three parameters, a state must make a decision on how much it needs to hold in reserve on an annual basis. In many respects, such a decision is analogous to determining the optimum level of water to be held in a reservoir. That is, the operator of a dam must specify the stock of water in a reservoir by releasing water from it. Given data on estimated inflows and outflows, the stability of such flows, and the benefits and cost of holding certain water levels, a "release rule" can be estimated which, in some sense, is optimal. Hence (by utilizing a benefit-cost approach in a stochastic setting), states can determine the optimum reserve level analogous to the example above. To this end, the model needs specific delineation.

The Optimum Model

As noted, a stock of reserves benefits a state by protecting it from the cost of adjusting expenditures and/or revenues in the presence of an imbalance in the budget of the state. Ultimately, the residents of the states must bear the burden or cost either in the form of higher taxes or lower levels of government expenditures. In this respect, a certain amount of income in the state is lost as a result of the multiple impact of tax increases and/or expenditure decreases when reserves are not available to meet contingent imbalances. Moreover, the benefits of state reserve holdings can be quantified as the product of reserves (equal to the deficit) times the income multiplier of the state. (The multiplier can be defined as one over the sum of the marginal propensity to save [MPS] and the marginal propensity to import [MPM]). That is:

\[
(1) \quad TB = \frac{1}{MPS + MPM} R
\]

The total benefits (TB) of reserve holdings can thus be measured by the ultimate income that is saved by holding reserves.

The measurement of the proper holding cost is somewhat more difficult. This emanates from the fact that controversy still exists over what the proper rate of return should be in evaluating the opportunity cost of a specific public sector policy. At one extreme, there are authors who contend that the discount rate should reflect society's time preference of foregoing present for future consumption. An alternative view is that the opportunity cost should be evaluated with a discount rate equilibrated to the social rate of return on capital. Unfortunately, no definitive work has prevailed on what the "proper" rate should be. In this paper, we have chosen to utilize the social rate of return on capital as the measure of the holding cost of reserves.

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3In fact, Nagabhushanam and Sastry used just such an analogy in discussing the demand for international reserves. See K. Nagabhushanam and M. Sastry [17].
4The analysis relies on the methodology of Heller in his 1966 study and the refinements made by Hipple in his 1975 work. See Heller [9], and Hipple [12].
5For further treatment, see J. Hirshleifer, et al [14], and W. J. Baumol [1].
In addition to specifying the proper rate of return, the measurement of holding cost needs to account for the fact that states hold their reserves in various types of interest-yielding government securities which have differing liquidity, divisibility, and reversibility characteristics. Thus, not only the social rate of return on capital, but also the composition and yield on short-term securities in which reserves are held need to be weighed in measuring the net cost of holding financial reserves. Given these requirements, the holding cost of reserves (TC) can be stated as:

**(2) TC = r(R)**

where \( r \) is the difference between the social rate of return on capital and the return obtained from holding reserves in short term liquid securities. That is to say, most states maintain their cash balances in highly liquid short-term securities.\(^6\) This provides a reserve which acts not only as a buffer against the possibility of insufficient funds, but also engenders some rate of return. Thus, subtracting this return from the long run return on capital gives the proper holding cost rate.

Given Equations (1) and (2), the optimum reserve holding which maximizes benefits can be obtained for a nonstochastic world by equilibrating the derivatives of (1) and (2). That is, the optimum level of reserves is attained where

**(3) \( r = \frac{1}{MPS + MPM} \)**

Equation (3) dictates the optimum level of reserves if a deterministic world is predicated. However, the ubiquitous existence of uncertainty makes it necessary to add the third parameter to the model—the probability that the reserve unit will be used. To this end, it can be argued that the holding cost of reserves is known with certainty. On the other hand, it seems gratuitous to make a similar supposition relative to the benefits of reserve holdings. Therefore, a third parameter that integrates the probability that there actually will be a need for reserves must be incorporated. Designating \( P \) as such a variable accounts for this uncertainty and redefines total benefits as:

**(4) TB = P \frac{1}{MPS + MPM}(R) \)**

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6Such assets include all liquid securities held in the portfolio of a state. This might include Demand Deposits, Time Deposits, Certificates of Deposits (on one-month or three-month maturity), Short-Term U. S. Notes, Treasury Bills (one or three months), or Federal Agency Securities. In general, considerable variance exists among states in the types of short-term securities held. In another study, the authors did find that states showed a predilection for Certificates of Deposits and Treasury Bonds. See M. M. Hackbart and R. S. Johnson [5].
and yields an optimum solution of,

\begin{equation}
(5) \quad r = \frac{P*}{MPS + MPM}
\end{equation}

or in the more general form,

\begin{equation}
(6) \quad R* = f(P*) = f(r[MPS + MPM]).
\end{equation}

Hence, P when equated to \((r) (MPS + MPM)\) represents the optimum holdings.\(^7\) The next step is to convert P into some probability distribution.\(^8\) After such specification, the value of the optimum reserve stock, \(R^*\), can be found from the probability, \(P^*\), that the reserve stock will be exhausted.

Utilizing the Heller approach for the optimum stock of international reserve holdings,\(^9\) it can first be posited that: (1) the size of any possible imbalance in the annual budget of a state, \(h\), can be estimated from the average absolute change in reserves or the change in the differences between projected and estimated budgets from year to year; with (2) a probability of \(P(B)\) that a surplus or deficit will occur each year; and (3) the balance in the budget over several years can be approximated by a random walk distribution.

Thus, if we use the present as the initial reference, then the probability that the first block of reserves, \(h\), would be used is \(P(B)\); the second \((P[B])^2\); and so on. Over the long run, then, the probability is

\begin{equation}
(7) \quad P_j = (P[B])^j.
\end{equation}

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\(^7\) For a true optimum to exist, it is necessary and sufficient that the second derivative be negative. Moreover, if MPS and MPM are assumed stable, as well as \(r\), then for an optimum to exist, which minimizes cost, a decreasing probability distribution is necessary. Moreover, such a condition is extant in a random walk model.

\(^8\) In the international reserve literature, several measures exist for determining the probability of a reserve unit actually being needed. Kelly, for example, relates the probability to the variance in exports. See Kelly [15, pp. 655-60]. Heller assumes that the change in the stock of liquid assets (or reserves) is a random process that can be approximated by a random walk. The amount of the imbalance can consequently be represented by the amount of change that occurs in a state's reserve stock, \(h\). The probability that a state will need to use an amount of reserves, \(R_i\), is thus given by the probability of \(i\) consecutive deficits of size \(h\). See Heller [9, pp. 302-04]. Finally, Hipple relying on the Heller approach, incorporates the speed of adjustment within the framework developed by Clark, Kenen, and Yudin. He does this by employing the Koyck lagged-adjustment process. See Hipple [12, pp. 630-34], Clark [2, pp. 363-64], and P. B. Kenen and E. B. Yudin [16].

\(^9\) The treatment of the probability of needing reserves relies on Hipple's discussion of Heller's approach. See Hipple [12, pp. 630-31].
j represents an index which, as Hippie points out, integrates all the discrete blocks of reserves, each of size h, and Pj is the probability of each block being used. Thus, the j index value for an individual reserve unit Ri within one of the h blocks is

(8) \( j = R_i / h \).

Therefore, (7) can be alternatively expressed as:

(9) \( P_j = (P[B])^{R_i / h} \).

Since P is equal to (r) (MPS + MPM), the optimum as delineated in (5) can be redefined as:

(10) \( P = (r)(MPS + MPM) = (P[B])^{R / h} \)

or by taking the logarithm of (10), and solving in terms of R, the optimal level of state reserves becomes:

(11) \( R^* = \frac{\log((MPS + MPM))}{\log(P[B])} \).

The optimum specified in (11) is similar to that which Heller derived in his 1966 study and Hippie in his 1975 article for the optimum level of international reserves. Applying it to state reserves, Equation (11) lends itself quite usefully to estimating reserve optimums and, in turn, the reserve adequacy of a state. That is, by estimating r, MPS, MPM, P(B), and h, the level of reserves a state should hold can be ascertained. Then, using the ratio of the estimated optimum over the average holdings actually held, a reserve index for a state can be developed to attest its adequacy.

An Example of the Optimum Reserve Level

The usefulness of Equation (11) is derived from its direct application for estimating the optimum reserve level. Only five parameters--MPS, MPM, r, P(D), and h--must be specified in estimating the optimum level of state reserves.

\[ \text{[10]} \text{ibid.} \]

\[ \text{[11]} \text{Pagan and Hippie have argued that Heller's optimum reserve Equation (11) is overstated by 100 percent. They contend that the error emanates from converting (7), a step-function, into a smooth approximation formula, (9). To rectify this, they subtract a P(B) in logarithmic form from the log (r[MPS + MPM]) in the numerator. Such a proposal was rejected by Heller. See H. R. Heller [10]. For certain criticisms of the Heller approach, see A. R. Pagan [18]. As noted, Hippie also refines Heller's optimum by incorporating a lagged term in the equation. However, in applying this model to state reserves, it seems that the fact that states must always maintain a balanced budget, the speed of adjustment is not applicable in this case. Thus, the Heller method is used as opposed to Hippie's.} \]
To consider the application of (11) for assessing state reserve holdings, this section specifies plausible values to represent a state government. The first variable to estimate is the marginal holding cost of reserves—specifically the difference between the social rate of return on capital and the yield on short term assets. From a theoretical point of view, the yield on capital would be estimated by determining either the marginal efficiency of investment or the present discounted value. In application, an appropriate proxy for the social rate of return on capital is the long-term rate of return on government bonds. For determining the rate of return on short-term securities, it is necessary to weigh the return on each asset held in the portfolio of the state by the percentage share of the asset. Research indicates that state short term holdings consist principally of Certificates of Deposits, Treasury Bills, and Demand Deposits [5]. Choosing a long-term rate of 8 percent and a weighted short-term portfolio return of 5 percent yields an r rate of 3 percent.

As for measuring the benefits of holding reserves, several tools exist. Because of the importance of interregional trade for states, the value of MPM becomes the critical variable in measuring the marginal adjustment cost of holding reserves. A useful method to estimate the state MPM is to utilize a state input-output model to calculate the average propensity to import (APM). The MPM could then be derived by assuming equality of the APM and MPM. For one midwestern state, the input-output model estimate of the APM (MPM), was 0.243. As for estimating the MPS of a state, it could be assumed equal to the MPS of the nation, specifically 0.25.

Finally, regarding the probability of needing a reserve unit, it was found for one state that the projected budget balance was on the average overestimated by approximately $12,000,000 annually. Moreover, only twice in the last ten years was it necessary to use any outstanding funds. Thus, as way of example, we might choose P(D) = 0.2 and h = 12,000,000. Table 1 reiterates these values.

Thus, inserting them into Equation (11) leads to an optimum reserve level of $31.2 million. That is:

\[
R^* = \frac{h \log(r[MPS + MPM])}{\log P(D)}
\]

(12) \[12,000,000 \frac{\log(0.03 \cdot 0.493)}{\log 0.2} = 31,200,000\]

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12 Figure derived from the 1969 Kansas Input-Output Model. See M. J. Emerson, et al [4].
13 0.25 was the value estimated by Teigen in his econometric model for the U. S. See R. L. Teigen [19].
14 Figure derived by aggregating the difference in revenues and expenditures over a ten-year period for one midwestern state. Data source: Bureau of the Census, State Government Finance.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.03</td>
</tr>
<tr>
<td>MPS</td>
<td>0.250</td>
</tr>
<tr>
<td>MPM</td>
<td>0.243</td>
</tr>
<tr>
<td>$P(D)$</td>
<td>0.2</td>
</tr>
<tr>
<td>$h$</td>
<td>$12,000,000$</td>
</tr>
</tbody>
</table>
$31,200,000$ represents the optimum in this example. Moreover, such an optimum can be used to test the adequacy of reserves by comparing the optimum with the average reserve holding as an index of $31,200,000/R$. Thus, state reserves can then be evaluated as excessively large or insufficient depending upon the value of $R$.

**Conclusion**

The question of whether or not the existing level of state reserve is, or is not, adequate requires considerable study. The present paper contributes to answering that question by suggesting a method for defining reserves, estimating their optimum level, and showing how such an optimum can, in turn, be employed to test reserve adequacy. Obviously, considerable empirical work is required before a definitive answer can be advanced on reserve adequacy. In any case, it is hoped that this paper illuminates an area of future empirical inquiry.
REFERENCES


