THE REGIONAL ALLOCATION OF INVESTMENT: A NEOCLASSICAL MODEL

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Introduction

The problem of the regional allocation of investment between regions is that of allocating an investment fund between two regions so as to maximize some objective function such as national income. The models of Rahman [11], Intriligator [5], and Takayama [14] attempt to maximize the national income at the end of the planning period assuming a constant capital coefficient production function for each region. Such models lead to the regional concentration of investment since all of the investment fund is allocated to one of the two regions in every year of the planning period. If the model does not include restrictions on regional income inequality,[1] the regional concentration of investment can lead to unacceptable differences in regional income levels.

Domazlicky [2] shows that the inclusion of transportation costs in a model with a constant capital coefficient production function can lead to autarchic growth; that is, each region is allowed to retain its own internally generated savings. The possibility of autarchic growth increases as transportation costs between the two regions increase. Autarchic growth is likely to be more acceptable than the regional concentration of investment.

The purpose of this paper is to construct a model of the optimal allocation of investment between two regions which possess neoclassical production functions. A previous attempt to construct such a model is given by Datta-Chaudhuri [1]. Isard and Liossatos [6-10] use basically a neoclassical production function in their space-time development models which treat space and time in a parallel manner. A neoclassical model in which substitution between capital and labor in the production process is permitted would appear to be more realistic than a model in which capital and labor are assumed to be used in fixed proportions. If the normal assumption of positive, but diminishing marginal productivity of capital is made in a neoclassical model, it is expected that such a model would lead to the regional dispersion of investment. The regional dispersion of investment is defined here as a situation in which each of the regions receive at least part of the investment fund. The reason for expecting the regional dispersion of investment is that any region which has an initial advantage in the productivity of capital would eventually lose that advantage, assuming

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[1] Rahman [11] does include such restrictions in his model.

the planning period to be sufficiently long, as it accumulates capital relative to the other region. If the above expectation is correct, a central planning authority can attempt to maximize national income through the regional allocation of investment and avoid the unacceptable solution of concentrating all investment in one region as is done in the models of Rahman, et al. It may be possible, therefore, to avoid placing restrictions on the allocation of investment which attempt to control the extent of regional income inequality.

A Neoclassical Model

It is assumed that there are two regions, each of which possesses a linear homogeneous production function:

$$(1) \quad Y_i(t) = F_i[K_i(t), L_i(t)] \qquad (i = 1,2)$$

where Y_i is income in region i, K_i is the stock of capital in region i, and where L_i is the amount of labor which is employed in region i. It is assumed that each factor is necessary for production so that:

$$F_i(0,L_i) = F_i(K_i,0) = F_i(0,0) \qquad (i = 1,2)$$

The production functions are assumed to be everywhere continuous and differentiable. The partial derivatives of the functions with respect to K_i and L_i are assumed to be positive but diminishing:

$$\partial F_i/\partial K_i > 0, \quad \partial F_i/\partial L_i > 0$$

$$\partial^2 F_i/\partial K_i^2 < 0, \quad \partial^2 F_i/\partial L_i^2 < 0 \qquad (i = 1,2)$$

Since the production functions are assumed to be linear homogeneous, division of (1) by $L_i(t)$ yields:[2]

$$(2) \quad Y_i/L_i \equiv y_i = F_i(k_i,1) \equiv f_i(k_i) \qquad (i = 1,2)$$

where $k_i = K_i/L_i$, and where $f_i(k_i)$ is a function whose value gives output per capita and whose value depends only on the capital-labor ratio in region i, Solow [13]. It is assumed further that the production functions satisfy the boundary conditions given by Inada [4]:

$$(3) \quad \text{(i)}\ f(0) = 0 \qquad \text{(iii)}\ f'(0) = \infty$$

$$\text{(ii)}\ f(\infty) = \infty \qquad \text{(iv)}\ f'(\infty) = 0$$

Labor in both regions is assumed to grow at a constant rate, n:

$$(4) \quad L_i(t) = L_i(0)e^{nt} \qquad (i = 1,2)$$

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[2] In the following analysis, timescripts are omitted except where they are necessary in order to prevent confusion.

where $L_i(0)$ is the initial labor force in region i. In order to simplify computations, it is assumed that $L_1(0) = L_2(0)$. This assumption does not materially affect the conclusions of this paper. It is also assumed that labor does not migrate between the two regions. The sum of these assumptions is that the two regions have identical labor forces.

Savings in each region is assumed to be some constant proportion of regional income:

$$(5) \quad S_i(t) = s_i F_i(t) \qquad (i = 1,2)$$

where S_i is total savings in region i, s_i is the constant savings ratio of region i, and where $F_i(t) = F_i[K_i(t), L_i(t)]$. Savings in the entire economy is assumed to be continually equal to investment:

$$(6) \quad I = \dot{K}_1 + \dot{K}_2 = s_1F_1(t) + s_2F_2(t)$$

where I is national investment and where a dot over a variable denotes the derivative of that variable with respect to time. A region's share of the total investment fund is equal to:

$$(7) \quad \dot{K}_1 = u[s_1F_1(t) + s_2F_2(t)]$$

$$(8) \quad \dot{K}_2 = (1-u)[s_1F_1(t) + s_2F_2(t)]$$

where u is the proportion of the investment fund that is allocated to region 1. It is assumed that capital cannot be shifted to region j once it is placed in region i; therefore, $0 \leq u \leq 1$.

The goal of the planning authority is to maximize the national income at the end of the planning period, T, regardless of the distribution of the national income between the two regions:

$$\text{Maximize } Y(T) = F_1[K_1(T), L_1(T)] + F_2[K_2(T), L_2(T)]$$

subject to Equations (7) and (8) and subject to the restriction that $0 \leq u \leq 1$. The problem is to select $u^*(t)$ so as to achieve the maximization of $Y(T)$, where $u^*(t)$ is the time path of u that generates maximum $Y(T)$.

Using the Maximum Principle, form the Hamiltonian expression, Takayama [10, pp. 600-617]:[3]

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[3]This uses the fact that the objective function can be written as:
$$\text{Max: } Y(T) = \int_0^T [F_1(L_1,\dot{K}_1) + F_2(L_2,\dot{K}_2)]dt + F_1[K_1(0), L_1(0)] + F_2[K_2(0), L_2(0)]$$

$$H = F_1(L_1,\dot{K}_1) + F_2(L_2,\dot{K}_2) + p_1u(s_1F_1+ s_2F_2)+ p_2(1-u)\ (s_1F_1+s_2F_2)$$

where the p_i's are the costate variables and are interpreted as the demand prices of capital.

The optimal time path is given by:

(9) $\partial H/\partial u = (p_1-p_2)(s_1F_1+ s_2F_2) = 0$

which gives:

(10) $u^*(t) = 1$, if $p_1>p_2$

(11) $u^*(t) = 0$, if $p_2>p_1$

Necessary conditions for a maximum are:

(12) $\dot{p}_i = -\partial H/\partial K_i$, $t < T$, $(i = 1,2)$

(13) $p_i = 0$ $t = T$, $(i = 1,2)$

Performing the differentiation as indicated in Equation (12) yields:

(14) $\dot{p}_1 = -[p_1u + p_2(1-u)]s_1(\partial F_1/\partial K_1)$

(15) $\dot{p}_2 = -[p_1u + p_2(1-u)]s_2(\partial F_2/\partial K_2)$

Solving the system of differential equations which is given by Equations (14) and (15), and using (13) yields:

(16) $$p_1(t) - p_2(t) = \frac{s_1(\partial F_1/\partial K_1) - s_2(\partial F_2/\partial K_2)}{s_2(\partial F_2/\partial K_2)}\, p_2(t), \quad t < T$$

Equation (16) indicates that the crucial factor in determining the regional allocation of investment is $[s_i(\partial F_i/\partial K_2)]$, the familiar reinvestment quotient, Rahman [11] and Datta-Chaudhuri [1]. For every t, such that $0 \leq t \leq T$, the entire investment fund will be allocated to the region with the higher reinvestment quotient.

In the absence of a central planning authority, Solow [13] has shown that the change in the capital-labor ratio over time in a neoclassical model is equal to:

(17) $\dot{k} = sf(k) - nk$

In such a model, for equilibrium growth, $\dot{k}$ must equal zero:

(18) $sf(k^*) = nk^*$

where k^* is the equilibrium capital-labor ratio. Capital grows at the same rate as labor so that the portion of the output per worker which is saved is just sufficient to equip workers at the same capital-labor ratio. If the boundary conditions of Inada [4] are assumed, the existence and stability of k^* are guaranteed.

The condition for equilibrium growth in a two region economy is altered if a central planning authority allocates investment between regions so as to maximize national income at the end of the planning period. Consider region 1 whose increase in the capital stock over time is equal to:

$$(19)\quad \dot{K}_1 = u[s_1F_1(K_1,L_1) + s_2F_2(K_2,L_2)]$$

By the assumption $k_i = K_i/L_i$, it is apparent that:

$$(20)\quad K_1 = k_1L_1$$

Differentiating Equation (20) with respect to time and equating the result to Equation (19) gives:

$$(21)\quad \dot{K}_1 = L_1(0)e^{nt}\,\dot{k}_1 + nk_1L_1(0)e^{nt} = u[s_1F_1 + s_2F_2]$$

Dividing both sides of Equation (21) by $L_1 = L_2 = L(0)e^{nt}$:

$$(22)\quad \dot{k}_1 = u(s_1f_1 + s_2f_2) - nk_1$$

where $f_i = f_i(k_i)$. For equilibrium growth in region 1, $\dot{k}_1$ must be equal to zero:

$$(23)\quad u[s_1f_1(k_1^*) + s_2f_2(k_2^*)] = nk_1^*$$

A similar procedure would show that for equilibrium growth in region 2, $\dot{k}_2$ must equal zero, therefore:

$$(24)\quad (1-u)[s_1f_1(k_1^*) + s_2f_2(k_2^*)] = nk_2^*$$

If for values of $k_1 = k_1^*$ and $k_2 = k_2^*$, Equations (23) and (24) are fulfilled, then $\dot{k}_1 = \dot{k}_2 = 0$, and equilibrium growth obtains in regions 1 and 2. Both regions will continue to grow at the rate, n, as long as equilibrium growth occurs.

The values of $k_1 = k_1^*$ and $k_2 = k_2^*$ are necessary for equilibrium growth. But those values must also satisfy Equation (16) if national income is to be maximized. First, note that $F_i(K_i,L_i) = L_if_i(k_i)$ and, therefore:

$$(25)\quad \partial F_i/\partial K_i = \partial[L_if_i(k_i)]/\partial K_i = f_i'(k_i)$$

This allows Equation (16) to be rewritten as:

$$(26)\quad p_1(t) - p_2(t) = \frac{s_1f_1'(k_1) - s_2f_2'(k_2)}{s_2f_2'(k_2)}\,p_2(t), \qquad t < T$$

Along the optimal path, there must be equality between $p_1(t)$ and $p_2(t)$, which means:

$$(27) \quad s_1 f_1'(k_1) = s_2 f_2'(k_2)$$

along the optimal path. Therefore, for equilibrium growth that maximizes national income, k_1^* and k_2^* must satisfy Equations (23) and (24) as well as Equation (27). In the following section, the existence and stability of such values for k_1 and k_2 is investigated.

The Stability of the Neoclassical Model

There are several cases that could be studied, given various assumptions on the values of s_i, f_i, n and $L_i(0)$. Four cases are given in Domazlicky [3]. The case that is considered in this section is for the assumptions $s_1 = s_2 = s$ (equal savings rates), $n_1 = n_2 = n$ (equal growth rates of labor), and $L_1(0) = L_2(0)$ (equal initial labor forces). The production functions are assumed to be linear homogeneous, but they are different between the regions. It is also assumed that the regions experience equilibrium growth before national income is maximized by a central planning authority. This means initially $\dot{k}_1 = \dot{k}_2 = 0$.[4] Assume that the production functions are such that k_1^* is greater than k_2^*, where a starred capital-labor ratio denotes the value of k_i at an initial point of equilibrium. This case is depicted in Figure 1.

The planning authority is charged with the task of allocating investment between the two regions so as to maximize national income at the end of the planning period. Assume that $f_1'(k_1^*)$ is greater than $f_2'(k_2^*)$. With $s_1 = s_2 = s$, this implies $sf_1'(k_1^*) > sf_2'(k_2^*)$. By Equation (26), this means $p_1(t) > p_2(t)$. Therefore, $u = 1$ and region 1 receives the entire investment fund in the initial years of the planning period. The implementation of this policy causes k_1 to rise and k_2 to fall. The increase in k_1 over time (with $u = 1$) is by Equation (22):

$$(28) \quad \dot{k}_1 = s[f_1(k_1) = f_2(k_2)] - nk_1$$

while the decrease in k_2 over time is equal to:

$$(29) \quad \dot{k}_2 = -nk_2$$

Given the Inada boundary conditions and a sufficiently long planning period, T, at some time $t = \bar{t}$, values of $k_1 = \bar{k}_1$ and $k_2 = \bar{k}_2$ are reached such that $f_1'[\bar{k}_1(\bar{t})] = f_2'[\bar{k}_2(\bar{t})]$ and, therefore, $p_1(\bar{t}) = p_2(\bar{t})$. These values are shown in Figure 1 where the slopes of the sf_1 and sf_2 curves are equal.

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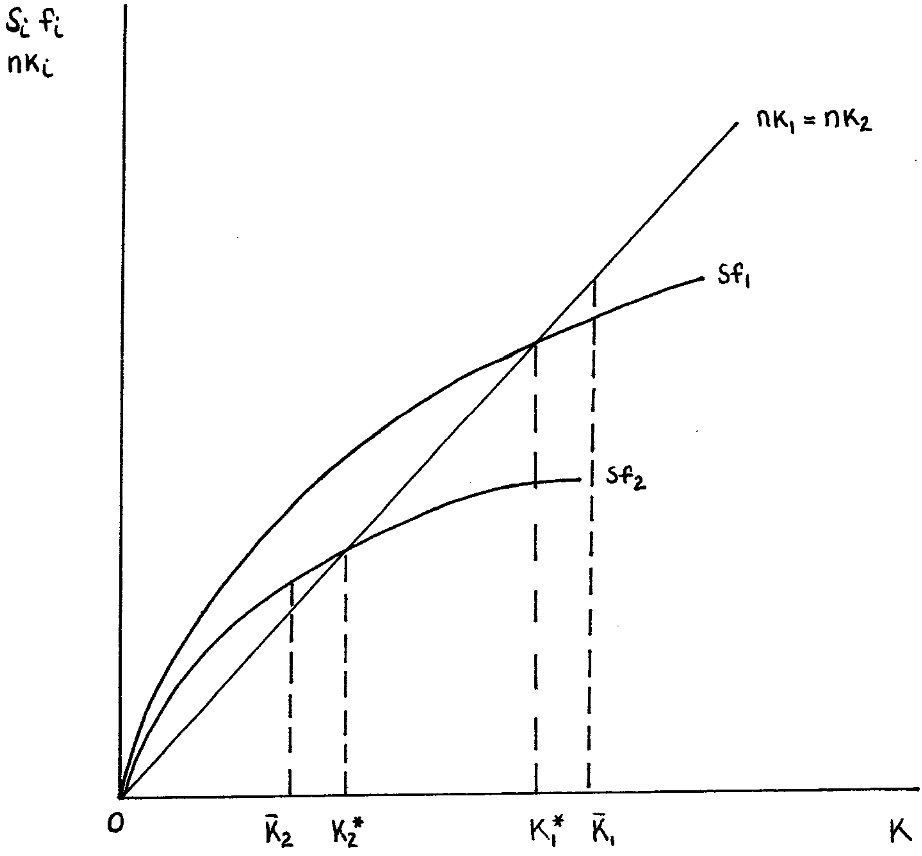
[4] The results of this section do not depend upon this assumption.

FIGURE 1: Different Production Functions



It might appear that from $\bar{t}$ until T, autarchic growth occurs. However, such a policy would cause $\bar{k}_1$ and $\bar{k}_2$ to change and, therefore, $p_1(t) \neq p_2(t)$. The program would not be experiencing optimal growth. Therefore, for $\bar{t} < t < T$, $p_2(t)$ must equal $p_1(t)$ for optimal growth. This implies $sf_1'(k_1) = sf_2'(k_2)$ for $\bar{t} \leq t < T$. At the combination $(\bar{k}_1,\bar{k}_2)$, $f_1'(\bar{k}_1) = f_2'(\bar{k}_2)$. However, whether this combination of k_1 and k_2 is maintained for $\bar{t} < t < T$ depends on whether Equations (23) and (24) are fulfilled. Adding (23) and (24) and using $\bar{k}_i$ instead of k_i* gives:

(30) $$s[f_1(\bar{k}_1) + f_2(\bar{k}_2)] = n(\bar{k}_1 + \bar{k}_2)$$

If Equation (30) holds, then $k_1 = \bar{k}_1$ and $k_2 = \bar{k}_2$ until T, the end of the planning period. Referring to Figure 1, this would mean that the over investment of region 2 at $\bar{k}_2$ just equals the under investment of region 1 at $\bar{k}_1$, and $\bar{k}_1$ and $\bar{k}_2$ will be maintained.

However, there is nothing to guarantee that $(\bar{k}_1,\bar{k}_2)$ will satisfy both Equations (27) and (30). This leads to a fundamental question: will there be values of $k_1 = k_1^A$ and $k_2 = k_2^A$ such that $f_1'(k_1^A) = f_2'(k_2^A)$ and $s[f_1(k_1^A) + f_2(k_2^A)] = n(k_1^A + k_2^A)$? If the two values of k_1 and k_2 do exist, will the program move to these values? If the answer to both questions is positive, then optimality is assured, $\dot{k}_1 = \dot{k}_2 = 0$, and equilibrium growth occurs in the two regions until T. The program will remain at (k_1^A, k_2^A) until the end of the planning period.

Consider first the equality $f_1'(k_1) = f_2'(k_2)$. Many pairs of values for k_1 and k_2 will equate $f_1'(k_1)$ and $f_2'(k_2)$. A line can be constructed such that all along that line, $f_1'(k_1) = f_2'(k_2)$. To construct such a line, take the total differential of $f_1'(k_1) = f_2'(k_2)$:

$$f_1''(k_1)dk_1 = f_2''(k_2)dk_2 \qquad \text{or}$$

(31) $$dk_1/dk_2 = f_2''/f_1''$$

which is positive since the second derivatives are assumed to be negative. The line $f_1'(k_1) = f_2'(k_2)$ is shown in Figure 2. The actual slope of the line depends, of course, on the values of the second derivatives. Points to the left and above the line $f_1' = f_2'$ are points where f_1' is greater than f_2'. Points to the right and below the line are points where f_1' is less than f_2'.

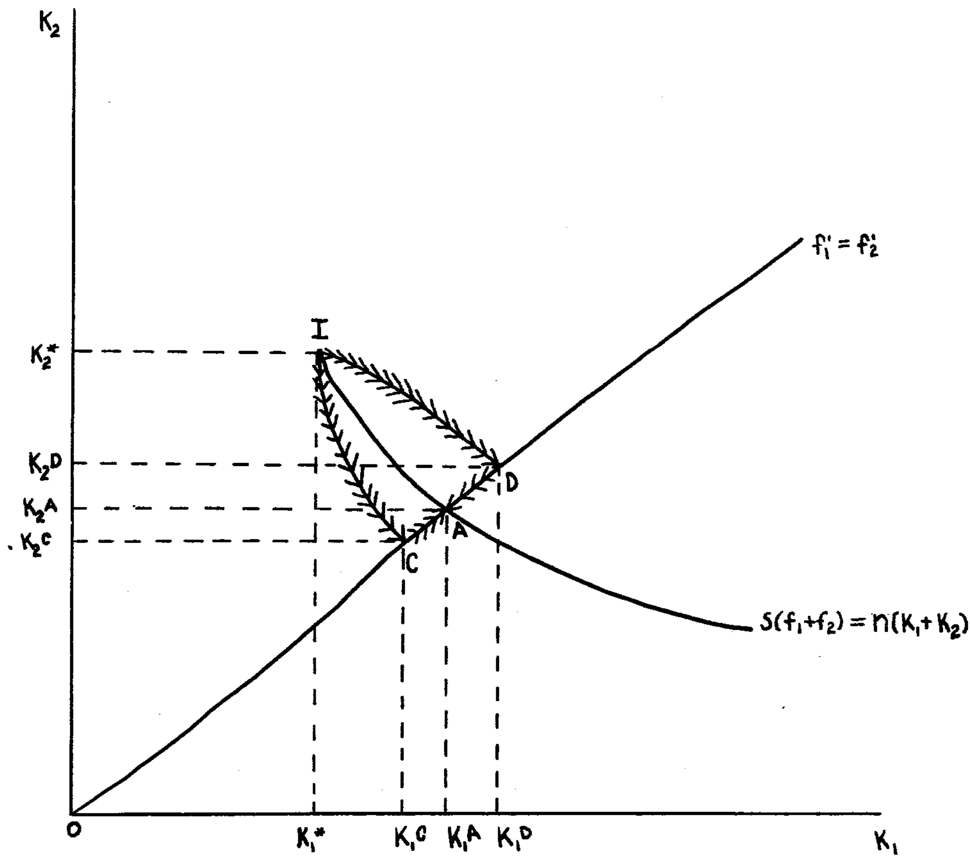
A line can be constructed such that all along the line $s(f_1 + f_2) = n(k_1 + k_2)$. Take the total differential of $s(f_1 + f_2) = n(k_1 + k_2)$:

$$sf_1'dk_1 + sf_2'dk_2 = ndk_1 + ndk_2 \qquad \text{or}$$

(32) $$dk_1/dk_2 = -(sf_2'-n)/(sf_1'-n)$$

Equation (32) is less than zero if both the numerator and denominator on the right hand side of the equation are negative (positive). For the initial

FIGURE 2: Equilibrium Capital-Labor Ratios When Income is Maximized



capital-labor ratios (k_1^*, k_2^*) to exist, we must have $sf_1'(k_1^*) < n$ and $sf_2'(k_2^*) < n$. Therefore, in the neighborhood of (k_1^*, k_2^*), the line $s(f_1 + f_2) = n(k_1 + k_2)$ is downward sloping. This is shown in Figure 2.

Point A is important to later analysis, therefore, it should be proven that $s(f_1 + f_2) = n(k_1 + k_2)$ is downward sloping at least to point A. At point I in Figure 2, $f_1'(k_1^*) > f_2'(k_2^*)$, which is the initial point for this case. This means:

(33) $sf_1'(k_1^*) - n > sf_2'(k_2^*) - n$

As the movement from point I to point A occurs, k_1 rises and k_2 falls. This means $f_1'(k_1)$ is decreasing and, therefore, $sf_1'(k_1)-n$ becomes more negative. At some point, equality is reached between $sf_1'(k_1)$ and $sf_2'(k_2)$. Therefore:

(34) $sf_1'(k_1) - n = sf_2'(k_2) - n$

However, both terms of Equation (34) are negative since $sf_1'(k_1) - n$ became more negative as k_1 rose. This means Equation (32) is equal to -1 at point A in Figure 2 where $f_1'(k_1) = f_2'(k_2)$. The slope of the line $s(f_1 + f_2) = n(k_1 + k_2)$ below point A is not relevant to this case, but it can be shown that the slope is negative for at least some (k_1, k_2) in that area.

It can now be stated that if the program moves to point A, optimal growth occurs at that point until the end of the planning period. The two regions also experience equilibrium growth (that is, $\dot{k}_1 = \dot{k}_2 = 0$) until the end of the planning period. To show that point A is the equilibrium point, recall Equation (9):

(9) $\partial H/\partial u = (p_1 - p_2)(sF_1 + sF_2)$

or that $p_1 = p_2$ along the optimal path. By Equation (26) this means $sf_1' = sf_2'$ along the optimal path.

At point I, $sf_1'(k_1^*) > sf_2'(k_2^*)$. As k_1 increases and k_2 decreases as a result of the maximization attempt, equality is reached at some time $t = \bar{t}$ between sf_1' and sf_2'.[5] For national income to be maximized, from $\bar{t}$ until T, this equality must be maintained. This means one of two cases must obtain at any time $\bar{t} \leq t < T$:

(35) $\dot{k}_1 = \dot{k}_2 = 0$

(36) k_1, k_2 increase (decrease) together so as to maintain equality between f_1' and f_2' until point A is reached. Once point A is attained, case (35) will obtain until time $t = T$.

Case (35) is apparent. If $\dot{k}_1 = \dot{k}_2 = 0$, then $f_1' = f_2'$ = a constant until T.

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[5] This assumes that the planning period is sufficiently long.

The over investment of region 2 is equal to the under investment of region 1. The program is at point A and will remain there for the remaining years of the planning period.

To show the latter assertion, consider Equation (36). All investment initially is allocated to region 1. As k_1 increases and k_2 decreases, f_1' falls and f_2' rises until at some (k_1^C, k_2^C) equality is reached between $f_1'(k_1^C)$ and $f_2'(k_2^C)$. This is shown as point C in Figure 2. At point C, however, $s[f_1(k_1^C) + f_2(k_2^C)] > n(k_1^C + k_2^C)$; the over investment of region 2 exceeds the under investment of region 1. There is sufficient total savings to maintain k_1^C and k_2^C and still have some savings remaining. The extra savings will be allocated between region 1 and region 2 so that k_1 and k_2 increase, maintaining equality between f_1' and f_2'. In Figure 2, this is indicated by the arrows along the $f_1' = f_2'$ line between points C and A. Once point A is attained, there are no extra savings available to increase k_1 and k_2. Until the end of the planning period, the program will remain at point A with $k_1 = k_1^A$, $k_2 = k_2^A$, $f_1'(k_1^A)$, and $f_2'(k_2^A)$, and $\dot{k}_1 = \dot{k}_2 = 0$.

Starting at point I as before, if equality between f_1' and f_2' is attained at (k_1^D, k_2^D) (see Figure 2), then $s[f_1(k_1^D) + f_2(k_2^D)] < n(k_1^D + k_2^D)$ and the over investment of region 2 is less than the under investment of region 1. There is insufficient savings to maintain (k_1^D, k_2^D). Therefore, k_1 and k_2 fall, maintaining equality between f_1' and f_2', until point A is reached. This is indicated by the arrows along the $f_1' = f_2'$ line between points D and A in Figure 2.

The conclusion is that along the optimal path, the equilibrium capital-labor ratios are k_1^A and k_2^A. These ratios obtain until the end of the planning period. Since $\dot{k}_1 = \dot{k}_2 = 0$, the value of u can be found as:[6]

$$(37) \quad u = k_1^A / (k_1^A + k_2^A)$$

Since k_1^A and k_2^A are both positive, $0 < u < 1$. Region 1 does not receive the entire investment fund. The regional dispersion of investment is the correct policy for the maximization of national income when a neoclassical production function is used. However, region 2 does send part of its savings to region 1. The amount of its savings that region 2 sends depends upon the explicit shapes of the production functions of the respective regions.

Summary and Conclusions

The models of Rahman, et al, that consider the regional allocation of investment lead to the regional concentration of investment. The reason for that result is the assumption of the use of capital and labor in fixed proportions. A region which has an initial advantage in the productivity of capital will tend to retain that advantage for the entire planning period.[7]

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[6]This uses Equations (23) and (24).

[7]However, under certain conditions, the program may switch from region i to region j.

If national income is to be maximized, the central planning authority must allocate the entire investment fund to one of the regions in every year of the planning period. If this result is unsatisfactory, then restrictions must be placed on the allocation of investment and, therefore, national income will not be maximized.

The use of a neoclassical production function leads to the regional dispersion of investment, given the assumptions of this paper. The regional dispersion of investment is a direct result of assuming a positive, but diminishing marginal productivity of capital. A region with an initial advantage in the productivity of capital eventually loses that advantage as it accumulates capital relative to the other region. There is some transfer of capital between regions, but a region does not send all of its capital to the other region. It is more likely that in a neoclassical model the central planning authority will be able to avoid placing restrictions on the allocation of investment. The maximization on national income is consistent with the regional dispersion of investment. The latter is likely to be more acceptable than the regional concentration of investment which occurs in the models of Rahman, et al.

In a free market economy, it is unlikely that a planner would have the authority to direct investment to the proper region as given by this model. However, market incentives (subsidies, taxes, etc.) could be used to insure that investment is placed in the proper region. Therefore, the model is useful in indicating the regional allocation of investment which is needed in order to maximize national income.

The model developed in this paper should only be considered an initial attempt at using a neoclassical production function to study the problem of the regional allocation of investment. Several limitations are apparent. The migration of labor is not allowed. As pointed out by Takayama [9], labor may be induced to migrate out of the region which receives no investment. Such migration may help to maintain the marginal productivity of capital in the region which receives the entire investment fund and, therefore, the result of the Stability of the Neoclassical Model may not hold. Second, the effects of agglomeration economies are not included in the model. If all of the investment is initially allocated to one region, this may permit economies of scale. As a result, that region's initial advantage may increase instead of narrow, Richardson [12, pp. 151-58]. Third, the model includes only one sector. Inclusion of additional sectors could be expected to increase the tendency toward the region dispersion of investment. A final criticism concerns the production functions. If the planning period is long, the assumption of unchanging production functions is untenable. Technical change especially may affect the marginal productivities of capital in the two regions and, therefore, change the regional allocation of investment.

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