

ASSESSING STATE RESERVE REQUIREMENTS: AN OPPORTUNITY
COST APPROACH

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In a recent article in Regional Science Perspectives [2], Johnson and Hackbart (JH) developed a cost-benefit approach to determination of the optimum level of financial reserves which a state should hold against the contingency that projected revenues are not realized. It is this author's contention that the conceptual definitions of both benefits and costs developed by JH were faulty. A correct conceptualization of the problem leads to a model which can better be used to estimate the optimal level of reserves.

JH state [2, p. 43] that "the benefits of state reserve holdings can be quantified as the product of reserves (equal to the deficit) times the income multiplier of the state," or

$$"TB = \frac{1}{MPS + MPM} R"$$

where

MPS = the marginal propensity to save
MPM = the marginal propensity to import
R = the level of reserves

It seems to this writer that the equation above is more logically the cost of holding a reserve in any period when it is not needed. Hence the expected cost of holding a reserve, R, would be:

$$(1) E(C) = \frac{1}{MPS + MPM} R(1 - P)$$

where

P = the probability that the reserve would be spent.

In other words, the cost of holding a reserve is the income foregone by not spending it.

JH also state [2, p. 44] that "the holding costs of reserves (TC) can be stated as:

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$$TC = r(R)$$

where r is the difference between the social rate of return on capital and the return obtained from holding reserves in short term liquid securities."

In the first place, r , as JH define it, would be negative whenever the government earned a higher interest rate on its reserves than the social rate of return. Since the social rate of return is usually measured by the current U. S. government bond rate [1, p. 150], which is among the lowest of all interest rates, it is not inconceivable that a state could frequently if not always earn a higher rate of interest on its reserves than the social rate of return. Under the JH cost specification, this would lead to negative costs of holding reserves and hence an infinite optimal level of reserves.

Further, on the conceptual level, it seems to this writer that the interest earned on reserves, times the multiplier (assuming that the interest earned is spent), is more logically a measure of the benefits of holding reserves. Hence the expected annual benefits of holding a reserve, R , would be:

$$(2) E(B) = \frac{1}{MPS + MPM} Rr(1 - P)$$

where r is simply the rate of interest earned on the reserves.

Unfortunately, if costs and benefits are conceived in this way, the costs of holding any level of reserves would always exceed benefits [since $r < 1$, equation (2) < (1)]. Thus if total income in a state is to be maximized, it is always better not to hold any reserves at all.

This finding is counterintuitive; however, as JH pointed out [2, p. 41], many states do in fact hold reserves. Therefore, they must not be attempting only to maximize total income.

Another benefit of holding a reserve is the political opportunity cost of not having a reserve when it is needed: the anger of state employees laid off, program clients cut off, and taxpayers facing tax increases. The expected annual value of this benefit can be written as:

$$(3) E_2(B) = AP$$

where A is the political opportunity cost of a revenue shortfall and P is the probability of a revenue shortfall.

The political opportunity cost, A , would be an increasing function of the level of reserves (i.e., the greater the level of reserves, the greater the benefit that would be enjoyed if the reserves were needed). If A were simply proportional to reserves, the net benefit function would be:

$$(4) E_1(B - C) = r_sR(1 - P) + cRP = sR(1 - P)$$

where

$$s = \frac{1}{MPS + MPM}$$

c = a constant which reflects the political opportunity cost weight attached to reserves; and

$$cR = A$$

Again in this case the optimum level of reserves is undefined. If the sum of the first two terms exceeds the third at one level of R , it will exceed it at any level of R ; so the optimum level of reserves is either zero or infinite.

If A increases faster than the level of reserves, an optimal level of reserves can be determined, but the result is unsatisfactory. For example, let

$$(5) E_2(B - C) = rsR(1 - P) + cR^2P - sR(1 - P)$$

where

$$cR^2 = A$$

Then

$$(5a) \frac{dE_2(B - C)}{dR} = rs(1 - P) + 2cR^1P - s(1 - P)$$

This first derivative can be set equal to zero and solved (given values of r , s , and P), but the second derivative is positive, so while a minimum exists, there is no level of reserves which maximizes net benefits.

If A increases at a slower rate than the level of reserves, a maximum can be found. For example, let

$$(6) E_3(B - C) = rsR(1 - P) + c\sqrt{R}P - sR(1 - P)$$

where

$$c\sqrt{R} = A$$

Then

$$(6a) \frac{dE_3(B - C)}{dR} = rs(1 - P) + \frac{cP}{2\sqrt{R}} - s(1 - P)$$

The second derivative of this function is negative, so setting the first derivative equal to zero and solving for R will yield the level of reserves which maximizes net benefits:

$$(6b) R = \left[\frac{c}{2(s-rs)} \cdot \frac{P}{(1-P)} \right]^2$$

This solution for the optimum level of reserves is intuitively appealing, too, in that R increases with increases in c (the political opportunity cost factor), r (the interest rate earned on reserves), and P (the probability of a revenue shortfall); and decreases with s (the income multiplier).

JH recognized that P is itself a declining function of R . Obviously, the larger the revenue shortfall, the smaller the likelihood that it would occur.

However, the particular function selected by JH ($P = (P[B]^{R/h})$, where $P[B]$ is the probability of using the first of a number of small "blocks" of reserves, and h is the number of such blocks--[2, p. 46]) cannot be utilized with the present model because if it is used the model cannot be solved.

The model can be solved if a function of the form $P = \frac{a}{\sqrt[n]{R}}$ where ($n > 2$; $R > a^n$; a and n are constants) is used. This function has a shape similar to the exponential used by JH: P declines toward zero asymptotically as $R \rightarrow \infty$. The requirement that $n > 2$ means that the rate of decline of P with R must be less (absolutely) than the rate of increase of A with R . This ensures that there is a maximum of the net benefit function, and that the maximum occurs at a positive value of R . In general, we define the net benefit function as:

$$(7) \quad E(B - C) = rsR(1 - \frac{a}{\sqrt[n]{R}}) + c \frac{k}{\sqrt{R}} \frac{a}{\sqrt[n]{R}} - sR(1 - \frac{a}{\sqrt[n]{R}})$$

where

- r = the rate of interest earned on the reserve
- s = the income multiplier
- R = the level of reserves, in millions of dollars
- a = a positive constant
- n = a constant $> k$
- c = a positive constant
- k = a constant > 1

The first derivative of net benefit function is given by:

$$(7a) \quad \frac{d E(B - C)}{dR} = \frac{n-1}{n} saR^{-1/n} (1-r) + \frac{n-k}{nk} caR^{(n-k-nk)/nk} + rs - s$$

The second derivative of this net benefit function is negative; hence, if we set (7a) equal to zero and solve for R , we will have the level of reserves which maximizes net benefits.

For example, let $s = 2$, $r = .05$, $c = 240,000$, $a = 11.247$, $k = 2$ and $n = 4$. Then the net benefit function is:

$$(8) \quad E(B - C) = (.05)(2)(R)(1 - \frac{11.247}{\sqrt[4]{R}}) + 240,000 \\ \frac{2}{\sqrt{R}} (\frac{11.247}{\sqrt{R}}) - (2)(R)(1 - \frac{11.247}{\sqrt[4]{R}})$$

The reader may verify that the level of reserves which maximizes net benefits is about \$30 million. Note that when $a = 11.247$, the probability of a \$10 million revenue shortfall is .20, which is similar to the JH example [2, p. 47].

A general solution for the optimum level of R is impossible, but if the

constants all meet the constraints listed after equation (7), there will always be a positive optimal level of reserves which can be found by trial and error.

The critical element in finding the optimal level of reserves is, of course, selecting the value of c used in equation 7 or 8. If the other parameters can be agreed upon, then levels of reserves actually held in different years by different states can be used to simulate implicit values of c . If the values of c appear to be consistent across time and across states for each given set of political conditions, then they can be applied in future determinations of the optimal level of reserves.

REFERENCES

1. Due, John F. and Ann F. Friedlaender. Government Finance, Sixth Edition, Homewood, Ill.: Richard D. Irwin, Inc., 1977.
2. Johnson, R. Stafford and Merlin M. Hackbart. "Assessing State Reserve Requirements: A Cost Benefit Approach," Regional Science Perspectives, 8 (1), (1978), 41-51.