GROUP PROGRAM EVALUATION FOR MULTIPLE OBJECTIVES

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Introduction

Many public programs are designed to attain economic and non-economic objectives (consequences) which may include higher real per capita income, lower unemployment rate, equitable income distribution, lower pollution, and/or a better state of public health, among others.

Traditional economic approaches tend to convert the non-economic objectives into monetary values or to neglect them entirely because they are difficult to convert into monetary values. The proposed approach attempts to encompass both economic and non-economic consequences and further attempts to treat the proposed program as a whole. To assist in this endeavor a two-stage decision making approach is developed which includes an optimal problem stage and a compromising selection stage.

Each decision maker is viewed as a maximizer for his own welfare and/or that of his constituents. The optimal problem process is a normative approach exploring how optimal program evaluation should proceed by an individual decision maker using the explicit value judgment. A compromising selection is viewed as a predictive approach for the expected result of a small group decision through modifications and compromises.

The worth of any program to an individual decision maker is the sum of its contributions toward achieving his weighted multiple objectives. Although an aggregation of the multiple objectives measured in heterogeneous units is necessary, they are incommensurable. In most practical cases, the decision makers may aggregate these weighted objectives implicitly without the aid of an explicit decision making process. However, any significant and complex decision requires explicit treatment of each decision making step. Furthermore, the future consequences and the resource requirements for the program are usually unknown without their objective probability distribution.

Although these problems are extremely complex, they should be incorporated into program evaluations, if the purpose of program analyses is to provide a useful guide for a decision maker.

The major sequence of a program evaluation in an optimal problem stage involves the following steps in each future time period for the relevent steps. (1) A set of mutually exclusive feasible programs are identified. (2) Plausible consequences corresponding to multiple objectives resulting from the alternative program are estimated. (3) Subjective probabilities to produce such consequences (objectives) are assigned. (4) The certainty equivalents of the expected values of the random consequences and their standard deviations are estimated. (5) The present value for the certainty equivalent of each con-

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sequence (objective) is calculated. (6) The marginal rates of indifferent substitutions among the present values of multiple objectives and their relative importance are estimated. (7) A worth indicator of each program is estimated. (8) The present values of resource costs for the alternative programs are estimated. (9) The worth-cost ratios for the alternative programs are estimated and the program with the highest worth-cost ratio is selected as the optimal program. (10) The compromising selection stage will be followed.

Optimal Problem Stage

For simplicity the following assumptions are made: three decision makers, three proposed programs, three objectives (consequences), two time periods, and three states of the world for each time period. The conclusions derived from these simple assumptions can be extended to more complicated and realistic cases.

The proposed program evaluation approach is limited to the cases where the decision makers or analysts have adequate knowledge for assigning subjective probabilities and such probability distribution assignments are plausible. The decision maker also must be willing to base his decision on the subjectively determined expected values and other derived values. Subjective probability is a measure of personal belief in outcomes based on his judgment, experiences, and incomplete information. Savage [4, 163-167] proved that the degree of personal belief can be measured numerically. However, the decisions are as reliable as the probability assignments to the random consequences and other subjectively derived values. The reason for relying on subjective probability is that objective probabilities for the future consequences based on observations and/or experiments are usually not available.

If the decision maker or analyst is coherent and unbiased, his subjective probabilities have the following three properties for consistency [4, 163-167].

Suppose S stands for a set of states of the world where S_1, S_2, and S_3 are subsets, then:

1. $0 \leqslant P(S_k) \leqslant 1 \qquad (k = 1, 2, 3)$
2. $P(S_1) + P(S_2) + P(S_3) = 1$
3. $P(S_1 \cup S_2 \cup S_3) = P(S_1) + P(S_2) + P(S_3)$, if and only if the states of the world are disjoint.

The decision maker's preference is described by a social welfare objective function which depends on the weighted sum of the social objectives. The social objectives are defined or grouped to be expressed as mutually exclusive and exhaustive consequences. The worth function is specified to be a linear and additive objective function as

$$(1) \qquad W = w_1 0_1 + w_2 0_2 + w_3 0_3 \qquad \text{where } 0 \leqslant w_j.$$

A linear approximation for relative importance within a reasonable interval over various magnitudes of objectives is assumed. The relative importance of each objective is explicitly stated and derived from the value judgment of the decision makers. When larger numbers of objectives are considered, some objectives may be equal to zero when a program does not change them at all.

Various levels of objectives (consequences) are usually expressed in heterogeneous units and they cannot be added directly. Fortunately, Van Eijk and Sandee [6, pp. 1-13] introduced an acceptable approach. They suggested that one of the weight coefficients be equal to unity and then derive the ratios of the coefficients without changing the value of the worth function W. If they do not meet the circular test of consistency for the relative importance, they recommended using the geometric average as the best approximates. The relative importance w_i can be derived by determining the equivalency between the magnitudes of the reference objective and other objectives. Suppose a decision maker's preference is such that $1,000 increase for real per capita income (0_1) is used as the reference objective and is perceived to be equal to:

1. 10,000 employment increase (0_2)
2. 5 percent decrease in air pollution (0_3)

Then, it is possible to derive the marginal rates of indifferent substitutions among multiple objectives. Using the equivalent relations, the estimated coefficient values for the trade-offs are as follows:

$$(2) \qquad W = w_1 0_1 + w_2 0_2 + w_3 0_3 = 1\, 0_1 + .1\, 0_2 + .2\, 0_3$$

In some cases the decision maker may not reveal the explicit equivalences between multiple objectives in order to avoid alienating diversified interest groups. When they are truly unknown to the decision maker, the analyst's role is to assist the decision maker to discover them by an iterative feedback procedure. Ideally, they should be explicit to avoid concealed value judgment and inconsistency.

The ratio of the worth indicator for the multiple objectives W to the discounted resource cost indicator R is called "the worth-cost ratio," W*. It represents the additional worth per dollar of the discounted resource cost estimated at shadow prices and represents the relative desirability of a given program. Consequently, the approach explicitly relates the alternative programs to their corresponding worth-cost ratios by using systematic procedures and explicit value system. (See Table 1).

The worth indicator W generated from a new program is the sum of the weighted contribution of each objective attainable. The additive representation theorem justifies an additive, order-preserving, real-valued representation as expressed in (1) [3, 3-26]. The worth indicator W is the numerical representation of the ordinal preference for an alternative program. The value of the worth-cost ratios should be based on the usefulness and reliability as they are used to assist the decision maker in choosing the optimal program.

The jth objective 0_j is a present value of the certainty equivalents of the jth consequences over time, discounted by the appropriate discount and/or time preference rates r_j. For the three objectives and two period example, the present value of the jth objective is

$$(3) \qquad 0_j = \sum_{t=1}^{T=2} \bar{\bar{C}}_j^t \,/\, (1 + r_j)^t \qquad (j = 1, 2, 3,)$$

The decision maker is assumed to be a risk averter who prefers higher expected consequence at lower risk, i.e. lower standard deviation SD. Since expected consequence is adjusted by a corresponding measure of its risk, it is possible to convert two parameters into a one dimensional certain future consequence. Tobin's justification for using indifference curves was that the decision maker could evaluate future consequences only in terms of two parameters; the mean and its standard deviation [5, 65-86].

TABLE 1. Alternative Programs and Worth-Cost Ratios

Alternative Programs	Relative Importance Consequences	Worth Indicators	Present Values of Costs	Worth-Cost Ratios
A_1	w_1 w_2 w_3 0_{11} 0_{12} 0_{13}	W_1	R_1	$W_1^* = W_1/R_1$
A_2	w_1 w_2 w_3 0_{21} 0_{22} 0_{23}	W_2	R_2	$W_2^* = W_2/R_2$
A_3	w_1 w_2 w_3 0_{31} 0_{32} 0_{33}	W_3	R_3	$W_3^* = W_3/R_3$

A_i — the ith alternative program
w_j — the jth relative importance
0_{ij} — the jth objectives generated by the ith program
W_i — the ith worth indicator
R_i — the present value of the ith resource cost
W_i^* — the worth-cost ratio for the ith program

The certainty equivalents may be used for the random consequences provided that the consequences are both mutually worth independent and probabilistically independent [2, pp. 276-287].

Worth independence describes the condition in which the relative preference for one objective does not depend on the specific amount of another objective [2, p. 286].

The certainty equivalent of the random consequences $\bar{\bar{C}}$ is defined as a function of the expected consequence $\bar{C}$ and its standard deviation SD [1, 14].

(4) $\bar{\bar{C}} = f\,(\bar{C}, SD)$

The expected value of finite discrete random consequences $\bar{C}$ can be estimated by assigning a subjective probability P(S) to each plausible random consequence C(S) at each mutally exclusive finite state of the world. (See Table 2 and 3).

TABLE 2. Alternative States of the World and Consequences Over Time.

	Social States at Time t=1			Social States at Time t=2		
States of the World	$(S=1)^1$	$(S=2)^1$	$(S=3)^1$	$(S=1)^2$	$(S=2)^2$	$(S=3)^2$
Probabilities	$P(S=1)^1$	$P(S=2)^1$	$P(S=3)^1$	$P(S=1)^2$	$P(S=2)^2$	$P(S=3)^2$
Alternative Programs	Random Consequences at Time t=1			Random Consequences at Time t=2		
A_1	$C(1)^1_{1,1}$	$C(2)^1_{1,1}$	$C(3)^1_{1,1}$	$C(1)^2_{1,1}$	$C(2)^2_{1,1}$	$C(3)^2_{1,1}$
A_1	$C(1)^1_{1,2}$	$C(2)^1_{1,2}$	$C(3)^1_{1,2}$	$C(1)^2_{1,2}$	$C(2)^2_{1,2}$	$C(3)^2_{1,2}$
A_1	$C(1)^1_{1,3}$	$C(2)^1_{1,3}$	$C(3)^1_{1,3}$	$C(1)^2_{1,3}$	$C(2)^2_{1,3}$	$C(3)^2_{1,3}$
A_2	$C(1)^1_{2,1}$	$C(2)^1_{2,1}$	$C(3)^1_{2,1}$	$C(1)^2_{2,1}$	$C(2)^2_{2,1}$	$C(3)^2_{2,1}$
A_2	$C(1)^1_{2,2}$	$C(2)^1_{2,2}$	$C(3)^1_{2,2}$	$C(1)^2_{2,2}$	$C(2)^2_{2,2}$	$C(3)^2_{2,2}$
A_2	$C(1)^1_{2,3}$	$C(2)^1_{2,3}$	$C(3)^1_{2,3}$	$C(1)^2_{2,3}$	$C(2)^2_{2,3}$	$C(3)^2_{2,3}$
A_3	$C(1)^1_{3,1}$	$C(2)^1_{3,1}$	$C(3)^1_{3,1}$	$C(1)^2_{3,1}$	$C(2)^2_{3,1}$	$C(3)^2_{3,1}$
A_3	$C(1)^1_{3,2}$	$C(2)^1_{3,2}$	$C(3)^1_{3,2}$	$C(1)^2_{3,2}$	$C(2)^2_{3,2}$	$C(3)^2_{3,2}$
A_3	$C(1)^1_{3,3}$	$C(2)^1_{3,3}$	$C(3)^1_{3,3}$	$C(1)^2_{3,3}$	$C(2)^2_{3,3}$	$C(3)^2_{3,3}$

$(S)^t$ - the Sth mutually exclusive state of the world at time t

$P(S)^t$ - a subjective probability for the Sth state of the world at time t

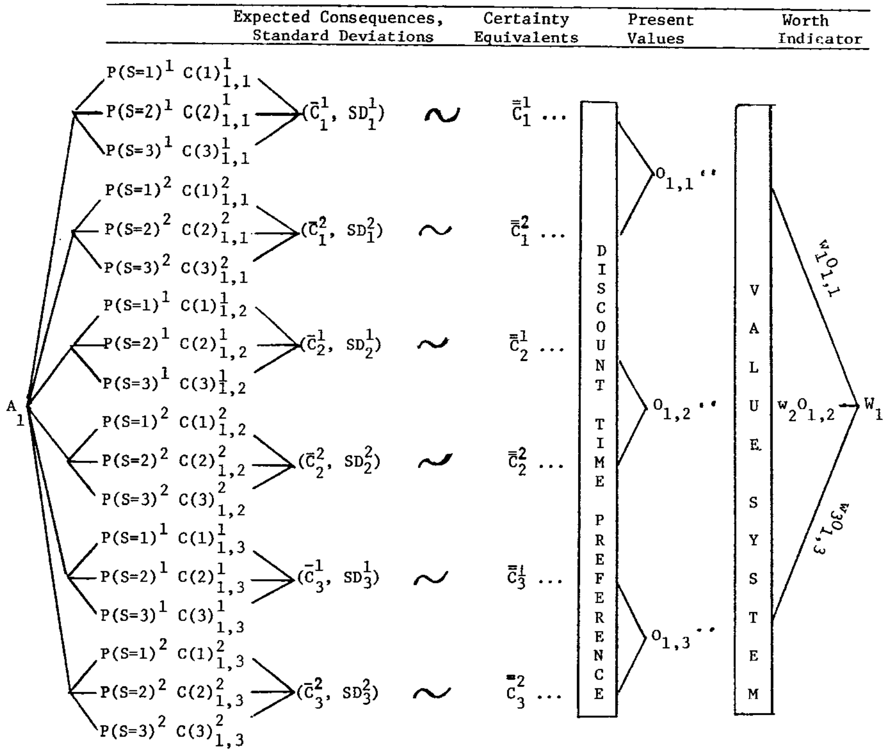
A_i - the ith proposed feasible program

$C(S)^t_{i,j}$ - the jth random consequence at time t that the ith program will yield if the Sth state of world occurs

$$(5) \quad \bar{C} = \sum_{S=1}^{3} P(S)\, C(S)$$

The expected value of the random consequences can be interpreted as the weighted mean of random consequence where the weights are the respective probabilities. Different sets of plausible probabilities may be used to estimate alternative expected values and their standard deviations in order to test their sensitivities.

TABLE 3. Program Evaluation Process For Program A_1



$P(S)^t$, $C(S)^t_{i,j}$	- see the footnotes in Table 2
$\bar{C}^t_j$	- the jth expected consequence at time t
SD^t_j	- the standard deviation for the jth consequence at time t
$\bar{\bar{C}}^t_j$	- the jth certainty equivalent for the jth consequence at time t
$O_{i,j}$	- the present value of the jth consequence from the ith program
w_j	- the relative importance of the jth objective
W_i	- the worth indicator for the ith program

$$(6) \quad SD = \sqrt{\sum_S [C(S) - \bar{C}]^2 P(S)}$$

The optimal choice problem is to choose a program with the highest worth-cost ratio W* subject to budgetary, technical, political and other relevant constraints at each period over the entire planning horizon. Since no other program generates a higher worth-cost ratio for the decision maker, it is the "optimal" program for his weighted multiple objectives.

Compromising Selection

When the highest worth-cost ratios are estimated by all decision makers, a group decision has to be made to select a program. The following assumptions are made for that purpose:

1. Each decision maker tries to improve or stay close to his worth-cost ratio acquired from the first stage optimal process for his own interest and/or that of his constituents.
2. The perference and voting decisions are based strictly on the merits of alternative proposed programs.
3. A simple majority rule is adopted as a decision rule.
4. Each decision maker possesses different magnitudes of the influencing capacity (clout) on the program evaluation at hand.
5. Three decision makers are assumed to avoid ties.
6. The decision at hand is neither related to any previous nor future program evaluation decisions.

Although three decision makers are confronted with the same three alternative programs, their worth-cost ratios may be entirely different due to various reasons. Since their subjective probabilities may not be the same, their expected consequences may be different. Even if their probability assignments were the same, their attitude toward risk may be different and thereby different certainty equivalents may be derived. Lastly, their marginal rates of indifferent substitutions between the objectives may be different. It is possible that an objective may carry a very high relative importance for one decision maker, but the same objective may be considered to have a small or even zero value by other decision makers.

If the probability of voting for the ith program with the highest worth-cost ratio is equal to one, i.e. $P(A_i) = 1$, then the probability of voting for the less preferable programs is equal to zero.

If all three decision makers derived the highest worth-cost ratios for the same ith program A_i, then it will be chosen by the unanimous votes. If two decision makers choose the same ith program, but the third person chooses one of the other programs, then the ith program will be chosen again. However, if each decision maker chooses a different program, then there is no majority solution.

Whenever a single majority vote is not reached for a given program, a possible solution is to modify a few carefully selected programs to make it acceptable to the majority. The modified program may be introduced by any decision maker or the analyst based on their known voting records, relative influencing capacities and worth-cost ratios.

The minimum required modification to he ith selected feasible program for the dth decision maker ΔA_{id} assures his positive vote for the modified program. The minimum required modification may be different for different decision makers. It may be explained by the potential relative influencing capacity of the d_{th} decision maker on the i_{ith} program I_{id}, the extent of its use in the i_{th} program E_{id}, and the difference between his highest estimated worth-cost ratio $\overline{W}_i^*$ and the worth-cost ratio of the proposed ith modified program $\hat{W}_{id}^*$

$$(7) \qquad \Delta A_{id} = f[E_{id}, I_{id}, (\hat{W}_{id}^* - W_i^*)] \quad 0 \leqslant E_{id} \leqslant 1, \ I_{id} \geqslant 0.$$

The minimum required modification for the dth decision maker is hypothesized as a positive function of each determining factor: E_{id}, I_{id}

and $(\hat{W}^*_{id} - \bar{W}^*_i)$.

For every proposed modified program, each decision maker has to re-calculate his worth-cost ratio. In evaluating each program, the decision making cost should be a factor and each decision maker should weigh the worth of his improved worth-cost ratio against his cost of re-calculating worth-cost ratio. If the new worth-cost ratio is higher than his highest estimated level or close enough such that the decision making cost does not justify another re-calculation, then he would cast a positive vote, otherwise a negative vote will result for a proposed modified program. These iterative procedures would continue until the decision group finds a compromised program with a single majority vote.

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