OPTIMAL ROADWAY TOLLS AND OPERATING COSTS IN URBANIZED REGIONS*

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Introduction

Roadway pricing has been frequently advocated to reduce traffic congestion during peak hours [8, 13, 14]. During peak hour use, added drivers increase vehicular concentration, reduce average speed and increase the opportunity costs of travel to fellow users. Unimpeded entry can increase traffic flow to a level where the social net benefits to all users are negative; the imposition of a toll equal to the time-delay costs imposed on others can result in an economically optimal vehicular flow. Drivers react to their own private costs of vehicular operation in deciding to traverse the roadway and not the social costs inclusive of the time delays imposed on others. This toll is a nonnegative, monotonically increasing function of traffic flow and is particularly applicable in high-income metropolitan regions where the time costs of commuting are substantial.

The traditional argument is silent as to how the private costs of operation per vehicle are determined.¹ If the driver’s choice of speed, acceleration and deceleration are not influenced by other vehicles on the roadway, then presumably no externalities are involved, and these costs are truly private. However, if other vehicles influence the maneuvers that a particular driver undertakes, then the observed private costs of operation are, in part, composed of an externality, or social cost, component. When one driver accelerates and changes lanes to avoid a speeding vehicle, the latter imposes an uncompensated external diseconomy on the former. Traffic engineers provide a model (termed “acceleration noise” and defined in equation (4) below) which depicts this vehicular interaction under alternative road conditions [1, 15, 16]. With a congested road, added drivers increase these accelerating and decelerating movements, so on a per-vehicle basis, the deviations from average speed increase. Engineers also suggest that these speed deviations are significant when the roadway is lightly traveled. If these vehicular interactions are reflected in operating costs, then the engineer’s theory provides a basis for analyzing deviations of private and social operating costs; the traditional congestion toll analysis must be supplemented with charges reflecting this interaction.²

¹ Segal [13, p. 175] presents the traditional textbook exposition of the congestion toll problem. Drivers pay an average cost to attain their destination which eventually rises as “car flow” increases. To this, we add the congestion costs imposed on others: no externality is considered in the vehicular operating costs due to drivers adjusting speeds to avoid accidents. References [10] and [11] provide more recent empirical applications of roadway pricing but do not deal with the basic engineering model that is utilized in this paper.

² The argument below rests only on the direct operating costs incurred when vehicular interaction occurs under high speed traffic conditions. Auto accidents may also be a cause of subsequently high levels of operating costs and medical expenses, job losses and reduced productivity from accidents; these costs are excluded here.
The purposes of this paper are to (1) examine the pricing implications of this engineering model for operating costs in conjunction with the traditional time cost component of the toll and (2) compare their relative significance from empirical data for a freeway section in Dallas, Texas. If the engineers are correct, we suggest that the optimal toll is understated at congested hours, if based only in the time-delay costs. Using data for the same freeway ensures comparability, and, unlike the usual estimation from historically averaged data, we use information from real-time experiments on freeway use. The policy issues include a growing concern with fuel conservation, rising gasoline prices and regulatory intervention during low-flow hours of freeway use; these are briefly discussed in the concluding section.

FIGURE 1. The Relationship Between the Fundamental Diagram and the Speed-acceleration Noise Model

Theoretical Model

The relationships among concentration, flow, speed, time and acceleration noise are graphically shown in Figure 1. The "fundamental diagram" is depicted in Figure 1(A) and is used in deriving the time-flow relation underlying the time-cost component of the toll. In part (B), acceleration noise is related to speed and converted to a time dimension for comparability to the fundamental diagram. The discussion in this section proceeds from a description of these physical relations to the economic variables defining the two toll components.
Vehicular Flow and Concentration. As shown by the fundamental diagram in
the southwest quadrant of figure 1(A), concentration (k) is a double-valued
function of flow (q). This equation is:

\[ q = k \cdot u \]

where \( q \) = the number of vehicles passing a point of roadway per unit of time,
\( k \) = the number of vehicles occupying a unit length of roadway at a point
in time, and
\( u \) = the “average speed” of the vehicles occupying a unit length of
roadway at a point in time.

Equation (1) depicts a variety of traffic conditions given the geometrics of a
particular roadway. Jam concentration \((k_j)\) is attained when vehicles are
solidly packed and the time delays imposed by added drivers are a ma-
maximum; as concentration falls from \(k_j\), \( q \) approaches a maximum (or critical)
flow \((q_m)\) with an associated speed of \(u_m(=q_m/k_m)\). Beyond this point, both \( q \)
and \( k \) approach zero as speed increases and approaches free speed \((u_f)\).3
Speed increases as concentration decreases from \(k_j\) to very small levels of
concentration; \( q \) and \( k \) are inversely related between \(k_j\) and \(k_m\) and directly
related for \( k \) between \(k_m\) and zero.

Following Walters [14] and Johnson [8], the time-flow and time-
concentration relations are derived from the fundamental diagram and
shown in the northwest and northeast quadrants, respectively, of Figure
1(A). Defining time \((t)\) as the reciprocal of speed, \(u_f\) determines the mi-
nimum trip time on a unit length of roadway for flow approaching zero. As
concentration increases, speed declines and trip time increases to the level
\( t_m \), consistent with \( q_m \). Beyond \( q_m \), increases in \( k \) reduce speed, and time
approaches infinity as \( k \) approaches \( k_j \). In the time-concentration quadrant,
the same minimum trip time is defined for concentration approaching zero
and \( t \) increasing, approaching the asymptote drawn at \( k_f \). Since equation (1)
may be written as \( q = k/t \), and the fundamental diagram has a unique
maximum flow at \( q_m \), it follows that \( k_m \) in the time-concentration quadrant is
defined by a point of tangency between the concentration-time function
and a ray through the origin. The southeast portion of Figure 1(A) is an
identity completing the principal relations derived from the fundamental
diagram.

To empirically estimate the parameters in equation (1), Greenshield’s [6]
linerization of the fundamental diagram is utilized in the form:

\[ u = u_f (1 - k/k_j) \]

3 The vector representation of \( u_f \) is excluded in the fundamental diagram but is the limiting speed in
the speed acceleration noise diagram in Figure 1(B). Behaviorally, \( u_f \) might be thought of as a
constant over a range of small, but positive, levels of concentration. Observed speed falls below
\( u_f \) when vehicular interaction becomes a consideration in the operator’s choice of speed. For our
purposes, a behavioral interpretation is preferable, since a lack of vehicular interaction may
characterize a range of concentration values, and the social costs imposed by drivers on each
other would be zero under these conditions.
This specification of the fundamental diagram suggests that if average speed is X% of free speed, then observed concentration is (1-X)% of jam concentration. The constraints implied in equation (2) suggest that \( k_m \) and \( u_m \) are 50%, respectively, of jam concentration and free speed; \( q_m \) is 25% of the product of \( k_m \) and \( u_r \).⁴

**Roadway Characteristics and Vehicular Interaction.** Figure 1(B) depicts the accelerating and decelerating responses of drivers to alternative average speeds of vehicles on the roadway. If only one driver uses the road, deviations from his average speed will usually be due to the physical characteristics of the road and the driving capabilities of the operator. When other vehicles are added to the roadway, a component of the average decelerating and accelerating responses of drivers is due to vehicular interaction; it is this interaction which causes the externalities that drivers impose on each other. As seen in Figure 1(B), the speed-acceleration noise model suggests that this interaction is substantial under both low and high average speed conditions.

As defined by Drew [3, p. 371], "total measured acceleration noise" is composed of two parts. "Natural noise" (\( \sigma_N \) in Figure 1(B)) is the minimum level of deviations from average speed and is due solely to the geometrics of the particular roadway section; the entire function may shift up or down depending on the driver’s skill, the quality of roadbed, number of lanes and road curvature. "Interaction noise" (defined as \( \sigma_I = \sigma - \sigma_N \), where \( \sigma \) is total measured acceleration noise) arises from stop-and-go conditions on the roadway section for very low speeds and from the rapid speed changes necessary to negotiate the traffic stream successfully at high average speeds.⁵ As shown in Figure 1(B), there is an optimal speed (\( u_m^* \)) at which \( \sigma_I = 0 \) and \( \sigma = \sigma_N \), so the total accelerating and decelerating responses of drivers are attributable only to physical characteristics of the roadway. If \( u(t_i) \) and \( a(t_i) \) denote the speed and acceleration, respectively, of a vehicle at time \( t_i \), then the average acceleration of this vehicle during a trip of duration \( T \) is:

\[
(3) \quad a_{ave} = \frac{1}{T} \int_{0}^{T} a(t_i) dt
\]

\[
= \frac{1}{T} [u(T) - u(0)]
\]

where \( u(0) \) = initial speed, and \( u(T) \) = final speed.

and according to Jones and Potts [9], total acceleration noise is defined to be:

\[
(4) \quad \sigma = \frac{1}{T} \int_{0}^{T} [a(t_i) - a_{ave}]^2 dt^{\frac{1}{2}}
\]

⁴ The somewhat lengthy derivation of these results are excluded for brevity. The development of these relationships is provided in Drinka [5, pp. 33-35].

⁵ Note that the unique minima of the acceleration noise function is an essential property in Figure 1(B). Exactly at \( u_m^* \), \( \sigma = \sigma_N \) since the absence of vehicular interaction means that only the roadway’s physical characteristics and the operator’s skill are responsible for changes in speed.
Figure 1 shows the translation of the speed-acceleration noise relation to a time scale. As above, time is the reciprocal of speed, so the speed-acceleration noise model is transformed to the time-acceleration relation in part (B) of Figure 1. As speed increases from zero, total acceleration noise \( \sigma \) approaches \( \sigma_n \) at speed \( u_m' \); for speeds greater than \( u_m' \), acceleration noise increases and approaches \( \sigma_{\text{max}} \) as \( u \) approaches \( u'_n \), so time falls and approaches zero. Due to the inverse relation between time and speed, a unique minimum time \( (t_m') \) is associated with the speed \( (u_m') \) at which \( \sigma_1 = 0 \).

Figure 1 also shows relationships among boundary values for the fundamental diagram and the acceleration noise model. Traffic analysts often use fluid flow theory to establish values of concentration, speed and flow utilized in the physical theory of roadway traffic. Drinka [5, pp. 53-61] formally demonstrates that maximal values of the two models do not, in general, correspond to each other. For example, the average speed \( (u_m') \) which minimizes total acceleration noise is fractionally larger than the critical speed \( (u_m) \) associated with equation (1): \( u_m' = 4/3 u_m \); and since \( u_m' = \frac{3}{2} u_n \), it follows that \( u_m' = \frac{3}{2} u_n \). Also, the corresponding levels of concentration \( (k_m') \) and flow \( (q_m') \) are fractionally smaller than the respective critical levels on the fundamental diagram; i.e., \( k_m' = \frac{3}{2} k_m \) and \( q_m' = \frac{8}{9} q_m \). Hence, in Figure 1 the average time of trip \( (t_m) \) corresponding to \( u_m' \) is less than \( t_m' \), and the time identity graph translates these minimizing time values into the respective values of \( q_m, q_m', k_m \), and \( k_m' \) on the fundamental diagram. The construction of Figure 1, therefore, reflects these interrelationships as derived from fluid flow theory. Empirical verification of the shape of the speed-acceleration noise model is given in Drinka [5, p. 105] and Drew, et. al. [4].

**Optimal Toll Schedules.** The physical model depicted in Figure 1 provides the basis for estimating the two components of the optimal toll. The time cost component is based on the flow-time function derived from the fundamental diagram, and the operating cost portion is estimated from the flow-operating cost function underlying the acceleration noise model. Both components involve a deviation of private and social costs, since vehicular interaction causes time delays and increases the costs of operating a vehicle by the erratic maneuvers necessary to avoid accidents. This deviation defines the optimal toll as the difference between the marginal and average social cost, where the latter are related to each other by:

\[
(5) \quad \text{MSTC} = (1 + \epsilon_1)\text{ASTC}, \quad \text{and}
\]

\[
(6) \quad \text{MSOC} = (1 + \epsilon_2)\text{ASOC}
\]

where \( \text{MSTC} \), \( \text{ASTC} \) = marginal and average social time cost,
\( \text{MSOC} \), \( \text{ASOC} \) = marginal and average social operating cost,
\( \epsilon_1, \epsilon_2 \) = elasticities of \( \text{ASTC} \) and \( \text{ASOC} \)

and the combined toll is the additive sum of both deviations. Figure 1 may be used to analyze the marginal and social cost deviations for both the time and operating cost components.

From Figure 1(A), increasing concentration implies additional driver interaction for speeds less than free speed. Beyond some minimum flow, divergences between ASTC and MSTC occur as additional drivers reduce the average speed of the entire vehicular stream. When flow is the measure of interest, ASTC approaches a maximum at \( q_m \) and continues to increase as flow
begins to decline with increasing concentration. At this point, rising ASTC is associated with declining flow, and MSTC is undefined; as Walters [14, p. 680] notes, one may assume that short-run marginal cost “is infinite for these levels of flow which are associated with a unit cost higher than that at” flow \( q_m \). Hence, the divergence of MSTC and ASTC is positive over most of the range of increasing flow, reaches a maximum at \( q_m \) and remains undefined for flow levels in excess of \( q_m \). These relationships are translated into a trip time dimension in the bottom graph of Figure 2.

From the acceleration noise model depicted in Figure 1(B), we may also infer the relation between MSOC and ASOC in equation (6) when flow is the measure of system output. When flow and concentration are virtually zero, free speed is reached and vehicular interaction is zero, so the private and social costs of operation do not diverge; that is, at free speed, all drivers can select their own desired speed and proceed unimpeded by other vehicles. However, as concentration and flow increase, speed falls and vehicular interaction occurs so that acceleration noise is substantial on a per-vehicle basis. Added drivers cause a reduction in average speed and acceleration noise, so MSOC is less than ASOC for a range of flows. Within this range, the added vehicle confers an uncompensated external economy on other drivers, for its presence reduces the fuel and braking costs necessary to avoid vehicular interaction. As speed is reduced below \( u_m \) in Figure 1(B), acceleration noise begins to rise and ASOC increases for flows increasing from \( q_m \) to \( q_m \). Here MSOC exceeds ASOC, and the positive operating toll reinforces the intent of the time cost component. As with the time cost part of the toll, the maximum possible divergence of MSOC and ASOC is defined at \( q_m \), and the definition of these relations is excluded when ASOC rises beyond this point.

The conversion of the physical to economic interpretations of Figure 1(A) and 1(B) vary slightly as between the time and operating cost components. The time cost portion is estimated from regressions relating flow to concentration. Speed changes are assumed to be solely due to variations in the number of vehicles on the roadway and are evaluated at the opportunity cost of travel. For the operating cost part of the toll, one must separate out the cost portion attributable to \( \sigma_1 \) and \( \sigma_N \). As an additional driver enters the roadway, only the portion of cost attributable to interaction causes the externality; the costs due to \( \sigma_N \) do not influence other drivers, so no externality is involved. Since our observations on operating costs include both components, we suggest a method for adjusting our data by subtracting out the “constant speed” component of operating costs. This method is described below, and equation (6) is subsequently interpreted as observed operating costs purged of costs due to \( \sigma_N \) (natural noise).

Figure 2 illustrates the two toll components and their relationship to critical trip time parameters. The time cost component increases as the trip time increases, and MSTC is undefined beyond \( t_m \), the time associated with maximum flow. The operating cost function is shown directly above the time cost graph in Figure 2. ASOC falls as trip time increases beyond \( u_m \) and then rises for trip times in excess of \( t_m \); the functional form is similar to the acceleration

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*Drinka [5, pp. 84-87] provides a discussion and graphical interpretation of these relationships using flow as the independent variable. On pages 117 to 120 of this reference, the acceleration noise-fuel cost relationship is analyzed.*
noise model graphed in Figure 1(B). From $u_f$ to $t_m'$, added drivers increase average trip time and reduce the operating costs attributable to vehicular interaction. For trip times greater than $t_m$, added drivers increase the average costs of vehicular interaction, so a toll will tend to discourage drivers entering the roadway.\(^7\)

\(^7\) Inter-urban income levels may be an important determinant of the differences in the nature of the combined toll. From Figure 2, higher income may increase the divergence of MSTC and ASTC, and the opposite results may be expected for regions with low incomes. The toll schedule may also be cyclically sensitive to income changes within given regions. Additionally, it should be noted that $t_m$ in Figure 2 may or may not be at the ASOC minimum when the latter is purged of fuel consumption not attributable to vehicular interaction. The new minimum will depend on characteristics of the "constant speed" fuel consumption equation, which may vary by type of vehicle and physical characteristics of the roadway.
Empirical Applications

Two types of data are required for estimating the toll components. First, speed and concentration observations are taken on a freeway section of the North Central expressway in Dallas, Texas, between the Fitzhugh off-ramp and the Mockingbird on-ramp. These samples were obtained for peak facility use on October 13 (a.m. and p.m.) and October 19 (a.m.), 1971. Second, additional data are taken from a moving vehicle study of an overlapping subsection of the same stretch of expressway. The operator of an instrumented vehicle is instructed to maneuver the auto through the designated section of the freeway in a normal manner that would maintain the pace of traffic. Three variables of interest are obtained from these data: mean velocity, acceleration noise and fuel consumption. These data were obtained between April 28, 1970, and November 16, 1970; there were no changes in the physical characteristics of the freeway section over the time period covered by the two sets of data. Most of the information used in this section was provided by the Texas Transportation Institute Urban Systems Program of Texas A&M. The following sections describe empirical techniques used to estimate the parameters underlying Figures 1 and 2.

Regression Estimates of the Physical Parameters. Both the time and operating cost components of the toll schedule are estimated by regression models. Since equation (1) fails to establish causal dependence, several alternative estimating forms are possible with the speed and concentration data. First, speed may be linearly dependent on concentration as specified in equation (2) above. The constant term is \( u_n \), and the slope coefficient equals \(-u_f/k_f\); thus, free speed, jam concentration and other critical values of the fundamental diagram may be estimated. Second, equation (2) may be multiplied by \( k \) to express flow as a quadratic function of concentration, since \( q = ku \) by equation (1). This equation has a zero constant term with linear and quadratic coefficients of \( u_f \) and \(-u_f/k_f\), respectively. For both specifications, three sets of data are available from the two dates (October 13 and 19), so six regression analyses were conducted, and single estimates of \( u_f \) and \( k_f \) were obtained. The following summarizes the results of these calculations; a more detailed discussion is provided in Drinka [5, pp. 87-96].

These six regressions produced highly significant statistical results with some variation in the estimates of boundary values in equation (1). All equational F-values were significant at a test level of \( \alpha = .01 \), and the lowest t-value on a variable coefficient was 4.332 in absolute value. All coefficients for \( u_f \) and \( -u_f/k_f \) were estimated with the correct sign. The variation in boundary values does not appear to be systematically related to time-of-day peaks in roadway use. Free speed, for example, ranged from 37.55 to 67.81 miles per hour among the six regressions, but both extreme estimates are calculated from the morning data. A somewhat similar variation occurs for estimates of \( u_m \); the high and low values are 31.69 and 18.77 miles per hour, respectively, with little consistency for the morning estimates. Though satisfactory statistical results are obtained for the six regressions, these variations suggest some difficulty in actually implementing the toll schedule.

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\* The entire section from Fitzhugh to Mockingbird Avenues is about 1.3 miles in length; the microscopic data covers subsystems of about 0.75 miles. These are deemed sufficient to attain accurate measures of the critical parameters. Drinka [5, pp. 79-84] provides a detailed discussion of the data, and Appendices B through D of this reference provide tabulations of the raw data and estimated cost schedules.
As both peak periods were available for October 13, and only the morning data were collected for October 19, the calculated tolls are based on regressions for the earlier date. The speed-concentration regression is converted to a time dimension by dividing its coefficients by 60 (minutes per hour). These regression results are summarized as:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Constant</th>
<th>Linear Coefficient</th>
<th>Quadratic Coefficient</th>
<th>F-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7) ( t = f(k) ) A.M.</td>
<td>1.110</td>
<td>-0.0104 (-5.577)</td>
<td>-</td>
<td>31.099 (7.56)</td>
</tr>
<tr>
<td>(8) ( t = f(k) ) P.M.</td>
<td>1.206</td>
<td>-0.0166 (-5.173)</td>
<td>-</td>
<td>26.761 (7.56)</td>
</tr>
<tr>
<td>(9) ( q = f(k) ) A.M.</td>
<td>0.0</td>
<td>67.8120 (21.376)</td>
<td>-0.6545 (-8.309)</td>
<td>1320.20 (5.39)</td>
</tr>
<tr>
<td>(10) ( q = f(k) ) A.M.</td>
<td>0.0</td>
<td>67.0444 (15.770)</td>
<td>-0.7603 (-4.332)</td>
<td>1463.70 (5.39)</td>
</tr>
</tbody>
</table>

where \( t \)-values appear below the slope coefficients, and \( \alpha \) values at the .01 level are below the computed \( F \) statistics. For this date, free speed ranges from a high of 72.36 to a low of 66.60 miles per hour, and the coefficient \(-uv/k_i\) ranges from -0.6240 to -0.9960; these range estimates are calculated from the time-concentration regressions by multiplying the constant and slope coefficients in equations (7) and (8) by 60.

The coefficient in equation (5), \( \varepsilon_1 \), is the elasticity of time with respect to flow or \( \varepsilon_{\text{a}} \). The latter equals \((\varepsilon_{ik} \cdot \varepsilon_{kn})\) or \((\varepsilon_{ik} \cdot \varepsilon_{ik}^{-1})\), so both of these coefficients may be obtained from equations (7) and (9) for the A.M. and equations (8) and (10) for the P.M. For different levels of concentration, we estimate \( \varepsilon_{ik} \) and \( \varepsilon_{ik}^{-1} \), multiply the two coefficients to get \( \varepsilon_1 \) in equation (5) and obtain estimates of MSTC. The time cost component is estimated at a $10 per hour wage rate with a $5 per hour evaluation of commutation time. With 1.5 occupants per vehicle. These estimates result in $0.125 per vehicle mile.

For the operating cost component of the toll, we have observations on acceleration-noise, fuel consumption and velocity for a single drive undertaking the runs described above. Drinka [5, pp. 56-61] provides an equational specification for the speed-acceleration noise relation shown in Figure 1. This equation has a constant term of \( \sigma_{\text{max}} \) (the intercept in Figure 1), a linear coefficient of zero and quadratic and cubic coefficients of \((-2/4)(\sigma_{\text{max}}/u^2)\) and \((27/4)(\sigma_{\text{max}}/u^2)\), respectively. If fuel consumption varies with acceleration noise, a similarly specified equation may be estimated by expressing fuel consumption (gallons per mile) as the dependent variable. These two regressions are:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Constant</th>
<th>Quadratic Coefficient</th>
<th>Cubic Coefficient</th>
<th>F-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(11) ( \sigma = f(u) )</td>
<td>2.1359</td>
<td>-0.1161 (10^{-2}) (-2.235)</td>
<td>0.1656 (10^{-4}) (1.998)</td>
<td>4.526 (4.31)</td>
</tr>
<tr>
<td>(12) ( f.c. = f(u) )</td>
<td>0.8125 (10^{-1})</td>
<td>-0.5859 (10^{-4}) (-5.674)</td>
<td>0.7593 (10^{-6}) (4.607)</td>
<td>59.240 (4.31)</td>
</tr>
</tbody>
</table>
where f.c. is fuel consumption, and all other coefficients are defined as those reported in equations (7) through (10) above. The acceleration noise regression has somewhat lower test statistics than equation (12); the equation’s F value (4.526) remains significant at the .01 level, but the quadratic and cubic coefficients are significant only at the .05 and .01 levels, respectively.

Parameters estimated from equations (11) and (12) show some variation despite acceptable statistical tests of significance. The minimum points on equations (11) and (12) occur at speeds of 46.7 and 51.4 miles per hour, respectively, for a difference of about 10%. It should be noted that the theoretical specification for the two regressions does not necessarily provide unique estimators of $\sigma_{\text{max}}$ and $u_t$. As $u_t = (-1.0) / \text{(cubic coefficient)}$ (quadratic coefficient), equations (11) and (12) provide free speed estimates of 70.1 and 77.1, respectively; these are virtually identical to estimates attained by multiplying $u_m$ by the theoretical constant of 3/2. (Note that from our definition of the coefficients, $u_t$ and $u_{\text{center}}$ as divisors in the quadratic and cubic coefficients. The estimates from $u_m$ are 1.5 (46.7) = 70.05 and 1.5 (51.4) = 77.1, from the equational minima cited above.) However, using the constant terms as estimates of $\sigma_{\text{max}}$, inserting these into the quadratic and cubic coefficients and solving for $u_t$ will produce several estimates of free speed. In equation (11), we attain $u_t$ estimates of 111.4 and 95.4 miles per hour, whereas in equation (12) our estimates are 96.7 and 89.7 miles per hour. (For example, $u_t = 111.4$ when $u_t(-.0016) = (-27/4)(2.1359)$ in equation (11)). The parametric constraints in our equations produce varying estimates of these boundary values.

The fuel consumption data will include both the effects of vehicular interaction and the amount of fuel consumed when the driver is not influenced by other vehicles on the roadway. Since the externality is attributable only to vehicular interaction, we would like to adjust our data for the noninteracting component of fuel consumed. Separate regressions based on data reported in the literature were estimated for this purpose, since our data do not include congested runs. The data in these regressions are attained for "constant speed" runs on roadways during congested hours. Six regressions were utilized; they specified fuel consumption as a quadratic function of speed with the following (high, low) parameters: constant term (0.0752, 0.0494), linear speed coefficient (-0.00072, -0.00209) and squared speed coefficient (0.000026, 0.000012). Variations in these parameters are due to differences in the vehicles used and road conditions under which these tests were performed. The 34 observations from these regressions are then pooled to estimate the adjustment equation:

$$
\begin{array}{ccc}
(13) & \text{Equation} & \text{Constant} \\
& f.c.' = f(u) & 0.6994 (10^{-1}) \\
& & -0.1224 (10^{-2}) \\
& & (-3.523) \\
& & 0.1818 (10^{+4}) \\
& & (4.258) \\
& & F = 6.41 \\
& & (4.41) \\
\end{array}
$$

The data for these regressions are taken from Figure 14-21 in [15, p. 342], Table 6 in [7, p. 17], Figure 13 in [2, p. 101], Figure 8 in [12, p. 43] and Figure 9 in [12, p. 43]. (Two regressions were run from reference [21]). These data result from runs conducted under controlled conditions, albeit for vehicles that have a substantial variation in ages. Reference [12] is perhaps most representative, resulting from runs conducted near Seattle, Washington, in 1963 on freeways unopened to the general public; the test vehicles were driven at speeds left to the discretion of the driver. The general shape of these functions are quite similar, suggesting that the major differences in fuel consumption are attributable to variations in constant terms in the regressions.
Equation (13) provides estimates of the two slope coefficients which are likely to be applicable to the Dallas data; the constant term, however, reflects the effects of different roadways and times that these tests were performed. Theoretically, equation (13) should be the lower envelope to equation (12), since fuel consumption due to interaction with other vehicles can only add to values predicted by equation (13). At least two procedures are possible. First, equate the first derivatives of equations (12) and (13) in the neighborhood of \(u_m\), solve equation (13) for an adjusted constant term. Subtracting the adjusted equation (13) from (12) results in an exact relation which has a zero first derivative on the speed axis. Second, subtract equation (13) from the 54 observations underlying equation (12) after adjusting the constant term in (13) so that 53 residuals are positive. This involves a search of the observations underlying equation (12) until a calculated minimum constant term in (13) is exactly consistent with one of the speed-fuel consumption observations. These differences become the dependent variable in a new equation which is structurally similar to equation (12).

The second procedure was followed, since evaluative statistics are available from the estimated regression equation. The minimum term becomes 0.037134 (instead of 0.06994 in equation (13)), and this new equation is consistent with an observation at 58.4 miles per hour with a consumption rate of 0.0277 gallons of gasoline per mile.\(^\text{10}\) The re-estimated regression is:

<table>
<thead>
<tr>
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<th>F-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(14) (f.c. = f(u))</td>
<td>0.6602 ((10^{-1}))</td>
<td>-0.534 ((10^{-4})) ((-4.324))</td>
<td>0.6046 ((10^{-6})) ((3.077))</td>
<td>F = 71.08 ((4.31))</td>
</tr>
</tbody>
</table>

Note the similarity of equations (12) and (14). The constant term is lower in equation (14) by 0.015 gallons per mile, and the quadratic coefficients in equations (12) and (14) are very similar. The cubic coefficient in equation (14) is slightly lower, but the evaluative statistics in equations (12) and (14) are quite comparable.

As with \(\varepsilon_1, \varepsilon_2\) in equation (6) is defined in terms of flow and, in addition, must be inclusive of the nonfuel costs of operation. From the speed-concentration versions of equations (7) and (8), flow may be expressed in terms of speed by substituting \(q \cdot u^{-1}\) for \(K\) and rearranging terms. Hence, for the October A.M. and P.M. periods, \(\varepsilon_{a1}\) may be calculated and multiplied by the elasticity estimated from equation (14), to yield \(\varepsilon_{f,c,a}\). The fuel cost component is estimated at $1.25 per gallon. Also, we have assumed that fuel costs are 50% of total vehicular operating costs. Note that the time and operating cost estimates approximately reflect current conditions; the physical characteristics underlying our regressions could be different under the real cost changes that have occurred over the past decade.

\(^{10}\) Note that the constant term is reduced by the computer search routine. The implication is that the car used in the Dallas Tests would have abnormally low fuel consumption along the same stretch of roadway under uncongested conditions. This may be due to the physical characteristics of the road or the use of a substantially more efficient test vehicle than was utilized in generating the "constant speed" data. Also, we have estimated the minimum of equation (14) at 58.8 miles per hour or about 7.4 miles higher than the minimum of equation (12). In the present case, our adjustment for "constant speed" fuel consumption shifts the equation slightly to the right.
Additional evidence regarding the consistency of the relation between the fundamental diagram and the acceleration noise model in Figure 1 is provided by our regressions. Since \( u_m' \) is theoretically equal to four-thirds of \( u_m \), estimates of \( u_m' \) from equations (7) through (10) should approximate optimum speed values from equations (11) and (12). From the latter regressions, we set the first derivatives equal to zero, to attain estimates of \( u_m' \) of 51.4 and 46.7 from equations (11) and (12), respectively. Multiplying \( u_m \) by four-thirds in equations (7) through (10) results in \( u_m' \) values of 44.4, 48.3, 45.2 and 44.7, respectively. Hence, the average \( u_m' \) calculated from equations (11) and (12) (i.e., 49.05) somewhat overestimates the values from equations (7) through (10). Note, however, that the overestimate is small being only about 3.4 miles per hour or approximately 6 to 7 percent of the critical values calculated from these regressions.

**Estimated Toll Components.** Table 1 shows the time and operating cost components of the toll for the morning peak of October 13 as estimated from our regression equations. Within the speed range of 40 to 55 miles per hour, the ratios of the operating cost to time cost component are 0.2537, 0.3717, 0.3498 and 0.2580 as speed decreases by 5-mile-per-hour increments from 55. The average ratio over those speeds is 0.3083, so the estimated operating cost component is not an inconsequential part of the combined toll.

It would be noted that due to the traditional concern with peak hour use, the data underlying the calculated tolls in Table 1 do not allow us to definitively estimate the nature of the schedule for off-peak hours. Hence, several of our parameters (e.g., \( u_t, \sigma_{\text{max}}, k_j \)) are generated from data observed during peak hour use. Alternative samples taken nearer to free speed conditions might reveal a substantially increased variance in estimated roadway parameters than would occur under peak-use sampling conditions. Since the engineering theory discussed here has implications for the high-speed hours of use, additional work might include samples for off peak use.

### TABLE 1: Estimated Toll Schedules for North Central Expressway, Dallas, Texas

<table>
<thead>
<tr>
<th>Column</th>
<th>(1) System Speed (m.p.h.)</th>
<th>(2) MSTC-ASTC (^a) (minutes per mile)</th>
<th>(3) Time cost component (^b) (cents per vehicle mile)</th>
<th>(4) MSOC-ASOC (^c) (gallons per mile)</th>
<th>(5) Operating Cost component (^d) (cents per mile)</th>
<th>(6) Combines toll (cents per mile) (^e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.441</td>
<td>5.513</td>
<td>0.0056</td>
<td>1.400</td>
<td>6.913</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.069</td>
<td>13.363</td>
<td>0.0199</td>
<td>4.975</td>
<td>18.338</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>2.678</td>
<td>33.475</td>
<td>0.0468</td>
<td>11.700</td>
<td>45.175</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>8.403</td>
<td>105.038</td>
<td>0.1084</td>
<td>27.100</td>
<td>132.136</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) Derived from Drinka [5, p. 111].

\(^b\) Column (2) times 12.5\% per vehicle-minute.

\(^c\) Derived from Drinka [5, p. 114] but purged of the constant speed component of acceleration noise.

\(^d\) Column (4) times $1.25 per gallon times two. The latter assumes that fuel costs are 50% of total vehicle operating costs.

\(^e\) Sum of columns (3) and (5).
Concluding Comments

In this paper we have examined some of the implications of roadway toll schedules estimated under the assumption that vehicular interaction causes two types of divergences between private and social costs. First, increments to concentration on a roadway impose time losses on the entire vehicular stream that are inversely related to average speed. This traditional component implies a toll schedule that is monotonically related to speed. Second, operating costs are increased by vehicular interaction both under low and high speed conditions on a per vehicle basis. The engineering theory suggests that stop-and-go conditions occur at highly congested periods with similarly high deviations from average speed when the highway is less congested.

We may also point to some of the policy implications of the theoretical models discussed in this paper. It seems likely that the relative importance of the two toll components is, if anything, understated by the data available to us. The increasing importance of fuel conservation and rising gas prices will increase the social costs of wasteful patterns of vehicular operation. The engineering theory of acceleration noise points to such patterns during the high-speed as well as low-speed hours of roadway use. Regulatory issues in the auto insurance industry are also related to high accident rates during lightly traveled periods of use. Of course, we may be skeptical that tolls based on the various types of externalities involved in roadway use will never be widely applied. Perhaps administratively less costly ways of regulation, such as rigorous enforcement of speed limits or staggered work hours during peak-hour use, would be more desirable. However, if roadway pricing is used, it should be recognized that the operating cost of vehicles involves an externality. This toll component seems likely to be of increasing significance in the future.
REFERENCES


