A Bayesian Spatial Econometric Analysis of SNAP Participation Rates in Appalachia

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Abstract. The Supplemental Nutrition Assistance Program (SNAP) is a federal program that provides assistance to low- and no-income people in the United States. Its aim is to increase individuals’ food-purchasing power and improve the nutritional content of their diet. We employed recent advances in Bayesian spatial econometric modeling to determine the appropriate model for drawing inferences about the percentage of SNAP recipients in Appalachia. We found that there is significant spatial dependence justifying the use of spatial econometric methods. We also examined how changes in an independent variable affect the dependent variable for orders of neighbors over space.

1. Introduction

The Supplemental Nutritional Analysis Program (SNAP) is the largest food assistance program for low-income households in the United States. The United States Department of Agriculture administers SNAP at the federal level, while state agencies administer the program at the state and local levels (USDA, 2010). States determine eligibility based on income and asset requirements before allocating and distributing benefits using an electronic benefit transfer system. SNAP is currently the cornerstone of the federal food assistance programs and catered to an average of 28.2 million people every month over the last 5 fiscal years (USDA, 2010).

The SNAP program can considerably improve a poor working family’s ability to purchase food. However, reports by the United States Department of Agriculture (USDA, 2011) show that not all eligible people take part in the program, recording a 60-70% participation rate over the last four years (Cunyngham and Castner, 2009). This has led to an increasing interest among researchers and governmental agencies in determining what influences individuals or households to participate in the SNAP program. Earlier studies on SNAP participation rates and caseloads were carried out at state and national levels using data from 1980 to 2004, with a majority employing Ordinary Least Square (OLS) and Feasible Generalized Least Square (FGLS) methods to conduct their analyses (Figlio, Gunder- sen, and Ziliak, 2000; Currie and Grogger, 2001; Kornfeld and Wilde, 2002; Kabbani and Wilde, 2003). A study by Goetz, Rupasingha, and Zimmer- man (2002) utilized spatial econometric methods to analyze participation in the food stamp program across the United States.

There is a paucity of studies analyzing the effects of social conditions on the SNAP program at the regional level. Furthermore, the use of Bayesian techniques in such studies has not been fully explored in the literature. This study examines the SNAP program at the regional level and controls for spatial autocorrelation. Spatial autocorrelation exists where the dependent variable or the error terms are correlated in a systematic manner over space. Ignoring this condition may lead to improper inferences because the coefficients estimated and standard errors may be biased, inconsistent, or both (LeSage, 1997). We formulated our model based on past studies and used Bayesian spatial econometric techniques to determine which variables affect participation in SNAP.
SNAP usage in the 417 Appalachian counties. We also addressed the model comparison issue in spatial econometric studies before selecting the preferred model and reporting results.

In the rest of the paper which follows, Section 2 gives a brief overview of the past literature, while section 3 develops the empirical model for analysis. Section 4 lays out the Bayesian econometric models. Section 5 presents the Bayesian spatial econometric models. Concluding remarks are given in section 7.

2. Literature review

Some studies have analyzed the supplemental Nutritional Analysis Program (SNAP) with regard to caseloads and participation. Studies by Figlio, Gundersen and Ziliak (2000), Goetz, Rupasingha, and Zimmerman (2002), Kornfeld and Wilde (2002), Kabbani and Wilde (2003), and Klerman and Danielson (2009) examined participation in the SNAP program by modeling recipients of the program, while Currie and Grogger (2001) and Ratcliffe et al. (2007), modeled receipts among households (Ribar and Edelhoch, 2008). Research on SNAP participation dynamics focused on nine broad issues which include participation patterns, economic activity, administrative and institutional policies, transaction costs, demographic factors, welfare policy and personal decisions.

The role and effectiveness of the SNAP program can be better understood by observing the changes taking place in participation patterns. The USDA (2010) reported that there were an additional 8 million SNAP participants in 2009 compared to those reported in 2005. There has also been concern over individuals who meet eligibility requirements but do not obtain benefits. On a monthly basis, statistics showed that 66.7% of all eligible individuals participated in the SNAP program in 2007 while 68% participated in 2006.

Changes in economic activities affect SNAP program participation because they affect employment opportunities and the number of working hours. Periods marked by economic downturns result in lower incomes and increased unemployment and poverty, thereby causing a rise in the number of households and individuals eligible to receive SNAP benefits (Hanson and Gundersen, 2002). Previous studies by Currie and Grogger (2001) and Wilde and Whitener (2001) found that reductions in SNAP participation caseloads were attributed to declines in unemployment, while Mossaad (2009) found that increased poverty led to a rise in SNAP participation. Studies conducted by Wilde et al. (2000) and the United States Department of Agriculture (2010) have shown that unemployment levels and SNAP program participation follow somewhat parallel tracks.

Transaction costs encompass the time taken to get to the SNAP office, time spent there, the burden of taking children to the office or paying for child-care services, missed hours of work and transport costs. To remain as participants, individuals periodically face these costs as they recertify their eligibility over intervals called recertification periods, which is a rule set by the federal government (Gundersen and Oliveira, 2001). The federal government requires that states recertify participants at least once a year although this may vary depending on the state. States adjust this period in a bid to keep up-to-date information on users and decrease error rates. Error rates refer to the over-payments or under-payments that the federal government makes to SNAP participants. In their paper, Kabbani and Wilde (2003) investigated the relationship between error rates, recertification periods, and SNAP participation and found that error rates had a significant effect on SNAP program participation. Past studies also found that using shorter recertification periods decreased SNAP participation either because ineligible participants were unable to participate or eligible participants failed to participate in the program (Currie and Grogger, 2001; Kornfeld and Wilde, 2002). Staveley, Stevens et al. (2002) found that the timing of SNAP exits in Maryland was clustered at the recertification dates. Lately, reporting practices in recertification among states have changed, allowing participants to recertify themselves over the phone, fax, or internet (Rowe et al., 2010).

Policies on welfare reform have the potential to alter the number of SNAP participants. This is due to the fact that persons receiving welfare are almost automatically eligible to receive SNAP benefits (McConnell and Ohls, 2001). For example, the Personal Responsibility and Work Opportunity Reconciliation Act of 1996 (PRWORA) changed welfare programs and altered eligibility rules for poor citizens. Adjustments in the program reduced participation in the SNAP program by tightening rules for able-bodied adults without dependents, who had to face a 3-month limit on receiving benefits unless they were working. PRWORA also made it harder for single mothers to receive cash welfare or SNAP benefits. Prior to the implementation of the 2002
Farm Bill, non-citizens could not receive SNAP benefits until they became citizens or worked in the country for a minimum of ten years (Wilde and Whitener, 2001). A study by Hanratty (2006) showed that SNAP program participation is highly correlated with a person’s age, parental race, education attainment, and disability status. Kim and Mergoupis (1997) argue that older and able-bodied individuals with higher incomes and fewer children are less likely to participate in the SNAP program. These are some of the issues illustrating the influence of institutional and demographic factors on SNAP participation.

Other factors such as stigma and lack of information can cause a decline in SNAP participation. Stigma refers to the distaste a person associates with SNAP participation and includes a wide variety of sources. It can stem from a person’s own repulsion for receiving SNAP benefits to his or her desire to avoid the embarrassment associated with being shunned by others when redeeming SNAP benefits. Other SNAP participants have a stigma associated with the possible negative reaction of caseworkers at the SNAP offices (Moffitt, 1983). McConnell and Ohls (2001) suggested that the stigma of accessing SNAP benefits in rural areas is lower compared to that in urban areas. The introduction of the electronic benefit transfer (EBT) system has reduced the stigma associated with the use of SNAP benefits but may make it harder for people unfamiliar with debit cards to get benefits (Currie and Grogger, 2001; Kabbani and Wilde, 2003). Although the system was introduced to reduce administrative expenses and discourage fraud, users of the card perceived less apprehension in using it in contrast to coupons used in the past. The system is currently in use in some states, while others are in the process of adopting it (USDA, 2011).

3. Methods and data

The dependent variable in our model is the Supplemental Nutritional Analysis Program (SNAP) participation rate. This variable is obtained by dividing the number of people who participate in the program by the total population in the county and multiplying this ratio by 100. For the independent variables, we employed variables that captured the influence of macroeconomic and business cycle conditions, individuals’ economic status, institutional factors, and demographics on SNAP participation.

The unemployment rate and employment growth rates are used to capture the influence of macroeconomic conditions in our model. We hypothesize that as the unemployment rate increases, the SNAP participation rate increases ceteris paribus. Unemployed individuals suffer a loss in income and may choose to enroll in the SNAP program to gain access to basic food support that may be lost during long periods of unemployment. The employment growth rate variable is designed to capture economic and business cycle conditions. It is also taken to be a measure of job availability, which is a more accurate measure of local economic conditions (Goetz et al., 2002). We hypothesize that this variable’s coefficient should have a negative sign, as growth in employment should be marked by a decrease in SNAP program participation, all else remaining equal.

To investigate the influence of a person’s economic status in his decision to participate in the SNAP program, we use poverty rate and non-labor income variables (Figlio, Gundersen, and Ziliak, 2000). We expect a positive relationship between the poverty rate and SNAP participation. The variable for non-labor sources of income captures household income which may include interest or dividends earned by individuals. Our expectation is that this variable should have a negative sign because non-labor income can be substituted for regular income. State recertification periods were included to study the effect of institutional factors and transaction costs on SNAP program participation (Kabbani and Wilde, 2003). The recertification interval is defined as the length of time that an individual has to wait to recertify as a legitimate participant in the SNAP program. It can be seen as a measure of the cost of SNAP participation to the individual because it entails the amount of time and money an individual has to put into following institutional requirements involving recertifying. The longer the recertification window, the lower the cost, so we expect that this variable will have a positive impact on the SNAP program participation rates.

Looking further into the effect of institutional factors on SNAP participation rates, state error rates were included to capture any anomalies in participation in the SNAP program. We have no a priori expectation regarding the sign of the error-rate coefficient variable. The enactment of the 2002 Farm Bill made it possible for legal immigrants to access SNAP benefits and we seek to investigate their role in SNAP participation. Consequently, the percentage of immigrants living in the county was included to explore the effect of demographics on the dependent variable. This variable has no a priori
expectation in terms of sign age. These expectations regarding the signs of the coefficient estimates for this particular set of covariates was motivated by previous studies and economic theory.

Following the discussion above, the general form of the model is expressed as follows:

\[ \ln \text{PART\_RATE}_c = \alpha_0 + \beta_1 \ln \text{UNEMP}_c + \beta_2 \ln \text{POVRTY}_c + \beta_3 \ln \text{NLINC}_c + \beta_4 \ln \text{RECERT}_c + \beta_5 \ln \text{ERRT}_c + \beta_6 \ln \text{IMMIG}_c + \epsilon \]

where the variable definitions are given in Table 1. In this model, the subscript \( c \) denotes the 417 counties in Appalachia. The data used for the empirical analysis were collected from various sources for the year 2007, with a summary of these statistics given in Table 2. In order to interpret our coefficients as elasticities, we transformed the dependent and some of the independent variables using natural logarithms. The independent variables which were transformed consist of the unemployment rate, poverty rate, recertification interval, error rate, and percentage of the population who are immigrants.

SNAP participation numbers, poverty rates and immigrant population data were obtained from the US Census Bureau, while data on employment growth rates was obtained from the Bureau of Labor Statistics. The Bureau of Economic Analysis provided data on non-labor sources of income while the Government Accountability Office provided data on error rates.

Table 1. Variables and corresponding abbreviations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PART_RATE</td>
<td>Supplemental Nutritional Analysis Participation Rate</td>
</tr>
<tr>
<td>UNEMP</td>
<td>Unemployment rate</td>
</tr>
<tr>
<td>EMPGR</td>
<td>Employment growth rate</td>
</tr>
<tr>
<td>POVRTY</td>
<td>Poverty rate</td>
</tr>
<tr>
<td>NLINC</td>
<td>Non labor Sources of income</td>
</tr>
<tr>
<td>RECERT</td>
<td>Recertification interval</td>
</tr>
<tr>
<td>ERRT</td>
<td>State SNAP participation error rate</td>
</tr>
<tr>
<td>IMMIG</td>
<td>Percentage of immigrants population</td>
</tr>
</tbody>
</table>

Table 2. Appalachian county-Level summary statistics (2007).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Number of Observations</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNAP participation Rate</td>
<td>417</td>
<td>0.1422</td>
<td>0.0708</td>
<td>0.0145</td>
<td>0.4932</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>417</td>
<td>5.4156</td>
<td>1.6754</td>
<td>2.3000</td>
<td>15.2000</td>
</tr>
<tr>
<td>Employment Growth rate</td>
<td>417</td>
<td>0.0869</td>
<td>2.3352</td>
<td>-10.2392</td>
<td>12.8839</td>
</tr>
<tr>
<td>Poverty Per Capita</td>
<td>417</td>
<td>17.5158</td>
<td>5.8659</td>
<td>5.2000</td>
<td>44.4000</td>
</tr>
<tr>
<td>Error rate</td>
<td>417</td>
<td>5.6924</td>
<td>2.4706</td>
<td>2.2300</td>
<td>9.5900</td>
</tr>
<tr>
<td>Non-Labor Income Per Capita</td>
<td>417</td>
<td>8.4436</td>
<td>4.4626</td>
<td>0.0000</td>
<td>21.1517</td>
</tr>
<tr>
<td>Percentage Immigrants</td>
<td>417</td>
<td>0.0866</td>
<td>0.1005</td>
<td>0.0010</td>
<td>0.9650</td>
</tr>
<tr>
<td>Recertification Periods</td>
<td>417</td>
<td>11.1281</td>
<td>3.2620</td>
<td>7.5000</td>
<td>19.0000</td>
</tr>
</tbody>
</table>

1 The employment growth rate and the non-labor income variables were not transformed using the natural logarithm because some of the observations contained the value zero or were negative.
3.1. Bayesian spatial econometric models and model comparison exercise

Given the geographic nature of the data, it is reasonable to suspect that spatial autocorrelation may be an issue. Spatial autocorrelation is formally defined as follows (Anselin and Bera, 1998):

\[
cov(y_i, y_j) = E(y_i, y_j) - E(y_i)E(y_j) \neq 0 \text{ for } i \neq j
\]  

(2)

where \( y_i \) and \( y_j \) are observations on a random variable at locations \( i \) and \( j \) in space. The subscripts \( i \) and \( j \) can refer to any geographic designation and the equation implies non-independence of the random variable across space. Spatial autocorrelation can pose problems when using standard econometric techniques, such as OLS.

Spatial econometric models come in three basic varieties, the spatial autoregressive (SAR) model, the spatial error (SEM) model, and the spatial Durbin (SDM) model. The spatial autoregressive (SAR) model can be represented as follows:

\[
y = \rho Wy + X \beta + \varepsilon
\]

(3)

\[
\varepsilon \sim MVN(0, \sigma^2 I_n)
\]

where \( y \) is an \( n \times 1 \) vector of observations on the dependent variable, \( X \) is an \( n \times k \) matrix of independent variables, \( \varepsilon \) is an \( n \times 1 \) vector of i.i.d., errors, \( \rho \) is a scalar spatial autocorrelation parameter, \( \beta \) is a \( k \times 1 \) vector of regression parameters, and \( W \) is an \( n \times n \) row-stochastic spatial weight matrix.\(^2\) It is possible that, for a variety of reasons, when an econometric model is specified and estimated certain factors that should be included in the model are not and that these factors are correlated over space, resulting in residual spatial error correlation. In our application regarding SNAP participation rates, there may be omitted factors such as cultural norms or other social phenomenon that are either not available or impossible to proxy for in any quantitative or qualitative sense. Additionally, it may be that the constituency boundaries cut across natural communities, resulting in spatial autocorrelation.

A failure to take account of this possible spatial dimension in the analysis means that if the true DGP is the SEM model and OLS is used as the estimation strategy, the OLS estimators of the coefficients are unbiased but inefficient and the estimates of the variance of the estimators are biased (LeSage and Pace, 2009). In practice, this can lead to incorrect inferences regarding the statistical significance of independent variables and thus lead to incorrect inferences regarding these independent variables.

A final spatial econometric model is a basic extension of the SAR model and labeled the spatial Durbin (SDM) model. The SDM is mathematically expressed as follows:

\[
y = \rho Wy + X \beta + WX \theta + \varepsilon
\]

(5)

\[
\varepsilon \sim MVN(0, \sigma^2 I_n)
\]

where \( y \) is an \( n \times 1 \) vector of observations on the dependent variable, \( X \) is an \( n \times k \) matrix of independent variables, \( \varepsilon \) is an \( n \times 1 \) vector of i.i.d. errors, \( \rho \) is a

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\(^2\)Technically, the spatial error model (SEM) illustrated in the text is a spatial error model with autoregressive errors. The other, less often used, SEM models is the spatial error model with moving average errors.
scalar spatial autocorrelation parameter, $\beta$ is a $k \times 1$ vector of regression parameters, $\theta$ is a $k \times 1$ vector of regression parameters on the spatially weighted $WX$ variables, and $W$ is an $n \times n$ row-stochastic spatial weight matrix. The difference between the standard SAR model described above and the SDM model is the inclusion of spatially weighted independent variables. LeSage and Pace (2009) suggest that the SDM model should be used when one believes that there may be omitted variables that follow a spatial process and are correlated with included independent variables. One such variable is that measuring the stigma associated with participating in the SNAP program. In certain areas, residents refrain from participating in the program due to the influence of social norms which “determine” the eligibility of participants. When the stigma associated with getting benefits is high, participants incur disutility in participating in the program (Bird, 1996). McConnell and Ohls (2001) found that participation rates were higher in the rural areas compared to urban areas because participants in rural areas had lower stigmas attached to SNAP participation. This is a geographical characteristic which is not controlled in this analysis and cannot be easily measured. It is also correlated with the included error rate variable which ultimately affects SNAP participation rates. In other words, we have a strong case that the spatial Durbin model is the most appropriate spatial econometric model to utilize in our analysis.

Our motivation for using Bayesian spatial econometric techniques, as opposed to the more familiar maximum likelihood paradigm, is that the Bayesian paradigm allows one to make non-nested model comparisons in a statistically coherent manner. Given this advantage, we now turn to the statistical development of the Bayesian variants of spatial econometric models and Bayesian model comparison.

By way of notation, let $\theta$ denote a vector of parameters of interest and $\pi(\theta)$ the prior probability density function (pdf) for $\theta$, and let $f(y|\theta)$ represent the likelihood function. The posterior distribution of the parameters, namely $\pi(y|\theta)$, is derived via Bayes’ Rule:

$$\pi(\theta|y) = \frac{\pi(y|\theta)\pi(\theta)}{\pi(y)}$$

(6)

where $\pi(y)$ is the integrating constant that ensures that the posterior probability density integrates to unity.$^3$ Given that $\pi(y)$ does not involve the parameter vector $\theta$, we can ignore this constant in subsequent analyses and write Bayes’ Theorem in a familiar form:

$$\pi(\theta|y) \propto \pi(y|\theta)\pi(\theta)$$

(7)

thus resulting in the familiar Bayesian phrase, “the posterior is proportional to the likelihood times the prior.” Ideally, we would like to draw inferences regarding the parameters of the model by analytically integrating the joint posterior distribution for each of the model’s parameters, resulting in a marginal distribution for each parameter. However, the analytical solution to this integration problem is available only in a few select cases. In deriving the marginal distributions, these complications force us to draw inferences using iterative procedures, referred to generically as Markov Chain Monte Carlo (MCMC) methods. Specifically, we will make use of the Gibbs sampler and the Metropolis-Hastings algorithm to provide robust inferences regarding the model parameters.

The Gibbs sampler is an algorithm used to generate a sequence of samples from the joint posterior distribution of the parameters when an analytical solution is unavailable. There are two necessary conditions for Gibbs sampling the SAR or SEM model, or any model, for that matter. First, the full conditional distributions comprising the joint posterior must be available in closed form. Second, these forms must be tractable in the sense that it is easy to draw samples from them.

The final full conditional distribution for $\rho$ is of unknown form so we must rely on the Metropolis-Hastings algorithm to draw inferences.$^4$ The Metropolis-Hastings algorithm is an accept-reject type algorithm in which a candidate value is proposed and then one decides whether to set the next value of the chain equal to this proposed value or to remain at the current value. The Metropolis-Hastings algorithm mimics the Gibbs sampling algorithm, but the difference is that the Metropolis-Hastings algorithm can be used for conditional distributions that do not have any recognizable distributional form. If the Metropolis-Hastings

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$^3$ The $\pi(y)$ quantity is also referred to as the marginal likelihood and plays a vital role in model comparison exercises.

$^4$ The full conditional distributions for the SEM model are derivable from its own joint posterior distribution for the parameters. The joint posterior distribution for the spatial Durbin model is the same as in the SAR case, with the exception that the $X$ matrix includes spatially weighted independent variables.
algorithm is used in combination with standard Gibbs sampling techniques, it is referred to as the "Metropolis-within-Gibbs" method. Further mathematical and computational details regarding MCMC estimation of spatial econometric models is covered in LeSage and Pace (2009) and Lacombe (2008).

The formula for Bayes’ Rule explicitly allows for prior information to be included in the statistical analysis. In each of our models we use proper prior distributions, but with relatively non-informative values. Specifically, we set the prior for \( \beta \) to come from a multivariate normal distribution with mean \( \hat{\beta} = 0 \) and covariance \( C_{\beta} = 10 \times I_k \). The prior values for the \( \sigma \) parameter, which comes from the inverted Gamma distribution, are \( \nu_0 = 1 \) and \( \alpha_0 = 1 \), and the prior value for the \( \rho \) term comes from a univariate normal prior with a mean of zero and standard deviation of 10,000. Similar values for the priors on the parameters were used for all of the other models estimated.

Another appealing aspect of Bayesian analysis is the formal statistical derivation of model comparison techniques. In the empirical application that follows, we were uncertain about which model is the correct one, i.e., SAR vs. SEM vs. SDM. We solve this problem by calculating posterior model probabilities and choosing the best model based on the calculation. The essential inputs in Bayesian model comparisons are the marginal likelihoods of competing models. As previously mentioned, the marginal likelihood, denoted \( \pi(y) \), is the integrating constant that ensures that the posterior distribution integrates to unity. Until recently, the computation of the marginal likelihood has proved to be extremely burdensome for all but the simplest models. In our model comparison exercise, we use the marginal likelihood calculation as outlined in Chib and Jeliazkov (2001), which is an extension of the algorithm proposed in Chib (1995) for models that include a Metropolis-Hastings step.

Model choice then proceeds by choosing the model with the highest value of the log-marginal likelihood. Table 4 contains the values of the log-marginal likelihood for each of three different spatial econometric models (SAR, SEM, and SDM). The model with the highest value of the log-marginal likelihood is the spatial Durbin model, and we therefore restrict our discussion in the results section to this specific model. We also note that we utilized a contiguity-based weight matrix \( W \) in each of the specifications. LeSage and Pace (2011) show that from both a theoretical and empirical perspective, the choice of which weight matrix to use is not as important as once believed in terms of drawing conclusions regarding the true effect of covariates on the dependent variable when these effects are properly calculated.

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Log-Marginal Likelihood</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAR</td>
<td>-35.334 (0.19665)</td>
</tr>
<tr>
<td>SEM</td>
<td>-6208.3 (0.19685)</td>
</tr>
<tr>
<td>SDM</td>
<td>-26.176 (0.19685)</td>
</tr>
</tbody>
</table>

Log-marginal likelihood values are calculated via the Chib and Jeliazkov method (Chib and Jeliazkov, 2001). Numbers in parentheses are the numerical standard errors of the log-marginal likelihood estimates.

4. Data analysis

Interpreting the way in which changes in the explanatory variables impact the dependent variable in spatial Durbin models is different than in a traditional ordinary least squares (OLS) model. In the case of the spatial Durbin model, the change in the dependent variable with respect to a change in the \( r \)-th explanatory variable takes the following form

\[
\hat{\beta}y' = \left( I_n - \hat{\rho}W \right)^{-1} \left( \hat{\beta} \hat{\beta}' + W \theta \right)
\]

where \( \hat{\beta}_r \) and \( \theta_r \) are the coefficient estimates associated with the \( r \)-th explanatory variable \( x_r \) and \( Wx_r \), respectively, and the \( \hat{\rho} \) term measures the strength of the spatial dependence. The expression in equation (8) results in an \( n \times n \) matrix of effects estimates. LeSage and Pace (2009) calculate point estimates using scalar summary measures of the diagonal and off-diagonal elements of this matrix expression. The direct effects are the averages of the diagonal elements and the indirect effects are the averages of the off-diagonal elements. The total effects are calculated as the sum of the direct and indirect effects.

The direct, indirect, and total effects estimates of the Bayesian Spatial Durbin Model are given in Table 4. The second column presents the direct effect estimates which relay the impacts of the independent variables on their own-county’s SNAP participation rate as well as feedback effects. The indirect effect estimates presented in the third column reflect the effects of changes in independent variables in
Table 4. Effects estimates results from the spatial Durbin model.

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Direct</th>
<th>Indirect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.4871</td>
<td>-0.3473</td>
<td>0.1397</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>[0.3423, 0.6315]</td>
<td>[-0.5964, -0.0957]</td>
<td>[-0.0745, 0.3556]</td>
</tr>
<tr>
<td></td>
<td>-0.0053</td>
<td>-0.0010</td>
<td>-0.0063</td>
</tr>
<tr>
<td>Employment Growth Rate</td>
<td>[-0.0165, 0.0056]</td>
<td>[-0.0315, 0.0300]</td>
<td>[-0.0390, 0.0266]</td>
</tr>
<tr>
<td>Poverty Percent</td>
<td>0.7329</td>
<td>0.8515</td>
<td>1.5844</td>
</tr>
<tr>
<td></td>
<td>[0.6043, 0.8606]</td>
<td>[0.6269, 1.0789]</td>
<td>[1.3653, 1.8136]</td>
</tr>
<tr>
<td>Error Rate</td>
<td>0.0417</td>
<td>0.0945</td>
<td>0.1362</td>
</tr>
<tr>
<td></td>
<td>[0.0712, 0.1549]</td>
<td>[-0.0591, 0.2465]</td>
<td>[0.0484, 0.2277]</td>
</tr>
<tr>
<td>Non-Labor Income</td>
<td>-0.0062</td>
<td>-0.0225</td>
<td>-0.0231</td>
</tr>
<tr>
<td></td>
<td>[0.0054, 0.0038]</td>
<td>[-0.0034]</td>
<td></td>
</tr>
<tr>
<td>Immigration</td>
<td>-0.0070</td>
<td>-0.0779</td>
<td>-0.0849</td>
</tr>
<tr>
<td></td>
<td>[-0.0387, 0.0238]</td>
<td>[-0.1412, -0.0153]</td>
<td>[-0.1496, -0.0226]</td>
</tr>
<tr>
<td>Recertification</td>
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<td>0.0260</td>
</tr>
<tr>
<td></td>
<td>[-0.0344, 0.3140]</td>
<td>[-0.3281, 0.1068]</td>
<td>[-0.1011, 0.1594]</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.2541</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.1754, 0.3324]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Effects estimates calculations based on 10,000 Gibbs draws with 10,000 draws allowed for burn-in. Quantities in brackets represent 95% credible intervals. Entries in the table in bold face are ones where the 95% credible interval does not contain zero.

neighboring counties on SNAP participation rate in own counties.\(^5\) The sums of the direct and indirect effects give the total effects estimates. These estimates reflect the variable’s effect on its own-county plus the (average) cumulative sum of impacts on all other counties as well (Kirby and LeSage, 2009).

As it is standard practice in Bayesian regression analyses, we calculated 95% credible intervals from the Gibbs samples for the regression coefficients. Those intervals that do not contain zero are considered ‘significant’ in the sense that the explanatory variable is associated with explaining variation in the dependent variable. The spatial autocorrelation coefficient, \( \rho \), has a 95% credible interval that does not contain zero, with a value of 0.25 indicating a moderate level of spatial autocorrelation in the dependent variable.

The direct and indirect effects estimates of the unemployment variable were associated with the dependent variable given the bounds on the 95% credible interval. A 10% increase in the unemployment rate increased SNAP participation rate by 4.87% within the county, whereas a similar increase in unemployment rate in neighboring counties reduced the county’s SNAP participation rate by 3.47%. Nevertheless, the total effects of the unemployment rate have no effect on SNAP participation rate, as the 95% credible interval contained the value zero. The direct, indirect, and total effects estimates of the employment growth rate had intervals that contained zero, thereby exhibiting no relationship between them and the dependent variable. The poverty rate exerted the greatest influence on SNAP participation rate in Appalachia due to the magnitude of the estimate for the total effect. A 10% increase in the poverty rate exerted a 7.33% increase in the SNAP participation rate within the county and a further 8.55% increase in SNAP participation stemming from the indirect effects. The overall effect is a 15.84% increase in SNAP participation rate.

The direct and indirect effects estimates of the error rate contained a zero and hence were not associated with the dependent variable. However, the total effect was associated, with 10% increase in the error rate leading to a 1.36% increase in SNAP participation rate. This did not come as a surprise since efforts to modernize the SNAP program in the region were underway and most states had not improved their systems, due to limited resources, unanticipated costs, or decreased staff resources as a result of the economic downturn. Therefore, error rates could not tell us much about participation. The non-labor income and recertification variables had

\(^5\) LeSage and Pace (2009) note that one can interpret the indirect effects in two different but numerically equivalent ways. One interpretation is that the indirect effect measures how a change in all elements of an explanatory variable affects one element of the dependent variable while the other measure how a change in a single element of an explanatory variable changes the dependent variable for all other units.
no effect on the participation rate, while the immigration variable exerted an indirect and total effect given the bounds on the 95% credible interval. A 10% increase in immigrant numbers in neighboring counties led to a 0.78% decrease in SNAP participation rate of own-counties and a 0.85% reduction in the total effect of Appalachia’s SNAP participation rate. Recent immigrants to Appalachia may be arriving at the request of other family members or may already have an alternative social network in place. Consequently, immigrants may not need the services of SNAP because they are relying on formal or informal social networks that provide for newly arrived members. Furthermore, they may not be aware of their eligibility for the program.

4.1. Impacts over space

In certain instances, we may be interested in how changes in an explanatory variable will affect the dependent variable based on the order of the neighbor relationship, i.e., how near neighbors are affected by a change in an independent variable versus how higher-order, or more distant, neighbors are affected.

The geographic scope of these effects can be shown by using the following matrix expansion:

\[
(I_n - \rho W)^{-1}(\beta_r + W \theta_r) \quad (9)
\]

\[
(I_n - \rho W)^{-1} = I_n + \rho W + \rho^2 W^2 + \ldots \quad (10)
\]

The effects falling on the first-order neighbors is given by \( \rho W(\beta_r + W \theta_r) \) for explanatory variable \( r \), while the effects falling on second-order neighbors are given by \( \rho^2 W^2(\beta_r + W \theta_r) \) for explanatory variable \( r \), and so on. Since the parameter \( \rho \) is less than one, we would expect to see a decline in the size of the effects as we move to higher-order neighboring counties (LeSage and Pace, 2009).\(^6\)

Table 5 contains information on impacts over space for one of our key variables, the poverty rate. The first column of Table 6 contains the order of the neighbor, where lower-order neighbors are defined as those that are close versus higher-order neighbors which are further away. For example, the row labeled "In" indicates the impact of the poverty rate on the own-county SNAP rate, while the "W" row indicates how the poverty rate affects the SNAP rate for the first order neighbors, the "W^2" indicates how the poverty rate affects the "neighbors-to-neighbors" and so on and so forth. The columns labeled "direct", "indirect", and "total" indicate how each of these effects changes over orders of neighbors. It should be noted that if we sum all of the individual effects over all neighboring relationships (i.e., we sum the columns) we would arrive at our effects estimates summaries for each type of effect.

Table 5. Summary of effects estimates for orders of neighbors.

<table>
<thead>
<tr>
<th>W-Order</th>
<th>Direct</th>
<th>Indirect</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>0.6916</td>
<td>0.4680</td>
<td>1.1596</td>
</tr>
<tr>
<td>W^1</td>
<td>0.0215</td>
<td>0.2964</td>
<td>0.3180</td>
</tr>
<tr>
<td>W^2</td>
<td>0.0112</td>
<td>0.0788</td>
<td>0.0900</td>
</tr>
<tr>
<td>W^3</td>
<td>0.0017</td>
<td>0.0245</td>
<td>0.0262</td>
</tr>
<tr>
<td>W^4</td>
<td>0.0005</td>
<td>0.0073</td>
<td>0.0079</td>
</tr>
<tr>
<td>W^5</td>
<td>0.0001</td>
<td>0.0023</td>
<td>0.0024</td>
</tr>
<tr>
<td>W^6</td>
<td>0.0000</td>
<td>0.0007</td>
<td>0.0008</td>
</tr>
<tr>
<td>W^7</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>W^8</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>W^9</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The spatially partitioned direct effect of poverty on SNAP participation is in column one of Table 5. We note first that the direct effect falls with increasing orders of neighbors, which makes intuitive sense in that the effect should decline as we move from lower to higher order neighbors. In the case of the poverty variable, we note a dramatic decrease in the direct effect with a value of 0.69 for the own-neighbor to 0.02 for the first-order neighbor. This pattern continues for the direct effect, and by the sixth-order neighbor the effect of a change in the poverty rate on SNAP participation disappears.

The pattern in terms of the decay of effect over space is similar in the case of the indirect effects, although the drop in magnitude is most pronounced at the second-order neighbors. In the case of the indirect effects, we find that these are negligible after about the ninth-order neighbor. A similar pattern holds for the total effects as well, in that by the ninth-order neighbor we see a negligible effects estimate.

\(^6\) LeSage and Pace (2009) provide additional details regarding the calculation and interpretation of these quantities.
These spatially-partitioned effects estimates give us a good sense of how changes in a particular independent variable, e.g., the poverty rate, affects SNAP in terms of geographic spread. The effect of poverty on SNAP participation would appear to be fairly localized in the case of the direct effects given the sudden drop-off in the value of this effect for different orders of neighbors. This should come as no surprise for two reasons. First, we would expect to see a drop-off in the value of an effect estimate over orders of neighbors simply because changes in an independent variable will affect the dependent variable more for closer counties than for those further away. Second, the value of our spatial autoregressive parameter from our spatial Durbin model estimates was approximately 0.25, which indicates a low amount of spatial autocorrelation in our dependent variable. Since the effects estimates are a function of the $\rho$ parameter (as well as other entities) we should not be surprised to see that the effect of changes in the poverty rate on SNAP participation is a relatively localized phenomenon.

5. Summary and conclusions

In this paper, we employed recent advances in Bayesian spatial econometric modeling to draw inferences about the factors affecting the SNAP participation in Appalachia. We used the marginal likelihood calculation outlined in Chib and Jeliazkov (2001) to determine which of the three spatial models to use. We found that the SDM model possessed the highest marginal likelihood value of -26.18 and hence used it for estimation and inference. Next, the direct, indirect and total effects estimates were calculated for the Bayesian Spatial Durbin Model, where a moderate level of spatial autocorrelation was reported. The results from the direct effect estimates revealed that only the unemployment and poverty variables were significant. The findings of the indirect effect estimates were similar, with the addition of the immigration variable as significant. With regard to total effect estimates, the poverty rate was found to exert the largest influence on the SNAP participation rate, while the immigration numbers had the least influence. The unemployment rate, employment growth rate, non-labor income, and recertification intervals were not found to influence SNAP participation. We also examined how changes in poverty affected the dependent variable for orders of neighbors over space and found that the effects estimates decayed fairly rapidly over space. The direct effects decayed to the sixth order while the indirect effects decayed to the ninth order.

Based on the research findings, our results present points of inference that can be useful for policy analysis in the Appalachian region. Specifically, the calculation of the direct, indirect, and total effects estimates can give additional information to policy makers in terms of how changes in policy affect not only citizens in their own counties, but how changes in policy can affect citizens in counties located in relative proximity. Relative to other econometric methods these estimates of effects in fact paint a more nuanced picture of how our control variables affect SNAP participation rates in Appalachia. Specifically, the direct effects show how an explanatory variable in a county affects SNAP participation in that same county, which is crucial for policy analysis. However, the total effects show how a change in an explanatory variable affects not only SNAP participation in the own county, but also in geographically-related counties. The implication is that policy makers may want to consider the spillover effects before implementing changes in SNAP policy.

In terms of practical application, planners and policy makers can use the research findings to forecast fiscal outlays during economic downturns. They can also ensure that eligible individuals with certain characteristics are not excluded from partaking in the SNAP program.

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